1. For each $\alpha$ and observed significance level (p-value) pair, indicate whether the null hypothesis would be rejected.
   
   (a) $\alpha = 0.05$, p-value = 0.10 \quad \text{fail to reject } H_0
   
   (b) $\alpha = 0.10$, p-value = 0.05 \quad \text{reject } H_0
   
   (c) $\alpha = 0.01$, p-value = 0.001 \quad \text{reject } H_0
   
   (d) $\alpha = 0.025$, p-value = 0.05 \quad \text{fail to reject } H_0
   
   (e) $\alpha = 0.10$, p-value = 0.45 \quad \text{fail to reject } H_0

2. In a test $H_0 : \mu = 100$ vs. $H_a : \mu > 100$, the sample data yielded a test statistic $z = 2.17$. Find and interpret the p-value for the test.

   \[
   \text{p-value} = P(Z \geq 2.17) = P(Z \leq -2.17) = 0.0150
   \]

   The chance of observing a test statistic of 2.17 or larger is only 0.015 or 1.5%.

3. An analyst tested $H_0 : \mu \geq 20$ against $H_a : \mu < 20$. The analyst reported a p-value of 0.06. What is the smallest value of $\alpha$ for which the null hypothesis would be rejected?

   \[ \alpha \geq 0.06 = \text{p-value} \]

4. Spending on housing. The Census Bureau reports that households spend an average of 31% of their total spending on housing. A homebuilders association in Cleveland wonders if the national findings apply in their area. They interview a sample of 40 households in the Cleveland metropolitan to learn what percent of spending devoted to housing goes toward housing. Denote by $\mu$ to be the mean percent of spending devoted to housing among all Cleveland households and note that the population standard deviation is $\sigma = 9.6\%$.

   (a) State the null and the alternative hypotheses.
   
   $H_0: \mu = 31$ \quad vs. $H_a: \mu \neq 31$

   (b) The study finds that the sample mean for 40 households selected at random is 28.6%. What is the value of the test statistics $z$?

   \[
   z = \frac{28.6 - 31}{9.6/\sqrt{40}} = -1.58
   \]

   Note that if you used percentages, you get $z = \frac{0.286 - 0.31}{0.096/\sqrt{40}} = -1.58$, same as before.

   (c) Sketch the normal curve and mark $z$ on the axis. Shade the area under the curve that represents the p-value; find the p-value.

   \[
   \text{p-value} = 2 \times 0.0571 = 0.1142 \approx 11.42\%
   \]
(d) Interpret the p-value in the context of the problem. Are you convinced that Cleveland differs from the national average?

The probability of getting a value for the sample mean as unusual as 28.6% when the population mean is 31% is 11.42%. This is not very unlikely, thus not too convincing evidence that Cleveland differs from the national average.

5. A survey of CPA's across the United States found that the average net income for sole proprietor CPAs is $74,914. Because this survey is now more than 10 years old, an accounting researcher wants to test this figure by taking a random sample of 112 sole proprietor accountants across the United States to determine whether the net income figure has changed. The sample yielded a mean income of $79,268. Assume the population standard deviation of net incomes for sole proprietor CPAs is $14,530.

(a) Test the researchers claim at the \( \alpha = 0.05 \) level of significance. Make sure to state a null and alternative hypothesis, a test statistic, p-value, decision and conclusion within the context of the problem.

\[ H_0: \mu = 74,914 \quad \text{vs.} \quad H_a: \mu \neq 74,914 \]

Test statistic: \[ Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{79,268 - 74,914}{14,530 / \sqrt{112}} = 3.17126 \]

p-value: \[ 2 \cdot P(Z \leq -3.17) = 2 \times 0.0016 = 0.0032 \]

p-value \( < 0.05 = \alpha \rightarrow \) reject \( H_0 \)

(b) How would the above test have changed, if the accounting researcher was interested in testing whether the net income has increased?

\[ H_0: \mu = 74,914 \quad \text{vs.} \quad H_a: \mu > 74,914 \]

Since \( Z = 3.17126 \Rightarrow \) p-value = 0.0016 \( \Rightarrow \) reject \( H_0 \) \( \Rightarrow \) we have statistically significant evidence that the net income figure increased.

(c) Explain what is the role of the sample size (\( n = 112 \)) in the validity of the results of this problem.

Since nothing is known about the shape of the distribution of the population of net incomes (we actually suspect that it is right skewed), we need a sample \( n > 40 \) to have a normal sampling distribution for \( \bar{X} \).

(d) Construct a 95% confidence interval for the present average net income of all sole proprietor CPAs.

\[ \bar{X} \pm z^* \frac{\sigma}{\sqrt{n}} \Rightarrow 79,268 \pm 1.96 \times \frac{14,530}{\sqrt{112}} = (76,610.34, 81,925.66) \]