

T-TESTS FOR POPULATION MEAN  $\mu$  (when  $\sigma$  is *unknown*)

1. **State Hypotheses:**  $H_0: \mu = \mu_0$   
 $H_a: \mu > \mu_0$   
 $H_a: \mu < \mu_0$   
 $H_a: \mu \neq \mu_0$

2. **Test Statistic:**  $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$

3. **p-value:** probability of getting more extreme test statistic than  $t$  given  $H_0$  is true

Under the assumption that  $H_0$  is true, the test statistic follows a  $t$ -distribution with  $df = n - 1$

$$H_a: \mu > \mu_0 \quad \text{p-value} = P(T > t)$$

$$H_a: \mu < \mu_0 \quad \text{p-value} = P(T < t) = \underbrace{P(T > -t)}_{\text{right area, easier for } t}$$

$$H_a: \mu \neq \mu_0 \quad \text{p-value} = 2P(T > |t|)$$

4. **Decision:** Reject  $H_0$  if

$$\text{p-value} \leq \alpha$$

5. **Conclusion:** In terms of the problem and  $H_a$

If reject  $H_0 \Rightarrow$  statistically significant evidence to support the claim made in  $H_a$

If fail to reject  $H_0 \Rightarrow$  *no* statistically significant evidence to support the claim made in  $H_a$

EXAMPLE 1: A random sample of 30 pills yielded a mean level of 20.5 mg of aspirin and a standard deviation of 1.5 mg. Assuming the pill is supposed to contain 20 mg of aspirin, are these sample pills acceptable at the 0.02 level?

1. State the null and alternative hypotheses:

2. Compute the value of the test statistic:

3. Find the p-value associated with the test statistic:

4. Decision:

5. There is \_\_\_\_\_ evidence (at the  $\alpha = 0.01$  level) to conclude that the pills have an average aspirin amount that differs from 20 mg.

EXAMPLE 2: A pain reliever currently being used in a hospital is known to bring relief to patients with a mean time of 3.5 minutes. A new drug administered to a random sample of 50 patients, yields a mean time to relief of 2.8 minutes and a the standard deviation of 1.14 minutes. Is there sufficient evidence that the new drug lowers the mean time to pain relief? Test at the  $\alpha = 0.05$  level.

1. State the null and alternative hypotheses:

2. Compute the value of the test statistic:

3. Find the p-value associated with the test statistic:

4. Decision:

5. There is \_\_\_\_\_ evidence (at the  $\alpha = 0.05$  level) to conclude that the new drug lowers the mean time to pain relief.