Z-Tests for Population mean μ (when σ is known)

1. **State Hypotheses:** H_0: $\mu = \mu_0$ versus H_a: $\mu > \mu_0$, H_a: $\mu < \mu_0$, H_a: $\mu \neq \mu_0$

2. **Test Statistic:**

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

3. **p-value:** probability of getting more extreme test statistic than z given H_0 is true

H_a: $\mu > \mu_0$ p-value $= P(Z > z)$

H_a: $\mu < \mu_0$ p-value $= P(Z < z)$

H_a: $\mu \neq \mu_0$ p-value $= 2P(Z > |z|) = 2P(Z < -|z|)$

4. **Decision:** Reject H_0 if

$$\text{p-value} \leq \alpha$$

5. **Conclusion:** In terms of the problem and H_a

If we reject $H_0 \Rightarrow$ statistically significant evidence to support the claim made in H_a

If we fail to reject $H_0 \Rightarrow$ *no* statistically significant evidence to support the claim made in H_a

EXAMPLES FOR TESTING A POPULATION MEAN μ, WHEN σ IS KNOWN
$(z - procedure)$

EXAMPLE 1:

Developing a new diet (to lose weight) and measure the # of lbs lost. Took a random sample of 36 people on the diet and found the average # of pounds lost to be 2.5 lbs. From past experience we know the population standard deviation to be 6 lbs. Is the diet effective? Conduct a test at the $\alpha = 0.01$ level of significance.

1. State the null and alternative hypotheses:

2. Compute the value of the test statistic:

3. Find the p-value associated with the test statistic:

4. Decision:

5. There is _____ evidence (at the $\alpha = 0.01$ level) to conclude that the diet is effective or mean weight loss is greater than zero.

Example 2:

A researcher believes that the mean score μ of all third graders in a district is lower than the national mean, which is 32. We know the scores are approximately normal with population standard deviation of 11 for this school district. Test the claim of the researcher at the $\alpha = 0.05$ level, using a random sample of 44 students in which the sample mean is 28.91.

1. State the null and alternative hypotheses:

2. Compute the value of the test statistic:

3. Find the p-value associated with the test statistic:

4. Decision:

5. There is _____ evidence (at the $\alpha = 0.05$ level) to conclude that the true score of all 3^{rd} graders in this district is less than 32 (National Average).

Example 3:

A manufacturer of a sprinkler system used for fire protection in office buildings claims that the true average system-activation temperature is $130°F$. A random sample of $n = 9$ systems when tested yielded a sample mean activation temperature of $131.08°F$. If the distribution of activation temperatures is normal with $\sigma = 1.5°F$, does the data contradict the manufacturer's claim at the 0.01 significance level?

1. State the null and alternative hypotheses:

2. Compute the value of the test statistic:

3. Find the p-value associated with the test statistic:

4. Decision:

5. There is _____ evidence to conclude that the true average activation temperature differs from the design value of $130°F$.