

Practice Problems

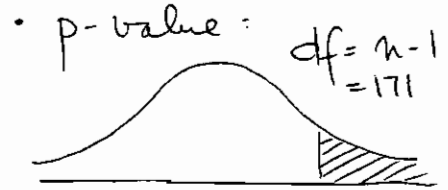
Use t-table
because σ is
unknown we only
have s .

A random sample of 172 marketing students was asked to rate on a scale from 1 (not important) to 5 (extremely important) health benefits as a job characteristic. The sample yielded a mean rating of 3.31 and a standard deviation of 0.70. Test at the 1% level of significance the claim that the population mean rating is bigger than 3.0.

$$\bar{X} = 3.31, s = 0.70, n = 172, \alpha = 0.01, \mu_0 = 3.0$$

$$H_0: \mu = 3.0 \quad \text{vs.} \quad H_a: \mu > 3.0$$

$$t_{\text{test}} = \frac{3.31 - 3.00}{0.70 / \sqrt{172}} = 5.80$$



$$p\text{-value} < \alpha = 0.01$$

\Rightarrow reject $H_0 \Rightarrow$ Statistically \Rightarrow Significant evidence at the 0.01 level to conclude that the pop. mean rating is bigger than 3.0

5.80 is off the chart, \Rightarrow p-value < 0.0005

2. It is important for airlines to follow the published scheduled departure times of flights.

Suppose that one airline that recently sampled the records of 246 flights originating in Orlando found that 10 flights were delayed for severe weather, 4 flights were delayed for maintenance concerns, and all the other flights were on time. Estimate the percentage of on-time departures using a 98% confidence interval. How would the interval change if you used Wilson's estimate?

$$n = 246, X = 246 - 14 = 232, \hat{p} = \frac{X}{n} = \frac{232}{246} = 0.9431$$

$$C = 98 \Rightarrow 1 - \alpha = .98 \Rightarrow \alpha = 0.02 \Rightarrow \alpha_2 = 0.01$$

$$\Rightarrow z^* = 2.33$$

a 98% CI:

$$\left[\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right] = \left[0.9431 \pm 2.33 \sqrt{\frac{0.9431(1-0.9431)}{246}} \right]$$

$$\Rightarrow [0.9086, 0.9775] \Rightarrow \text{length} \approx 0.0689$$

Wilson's estimate: $\tilde{p} = \frac{X+2}{n+4} = \frac{234}{250} = 0.936$

$$CI: \left[0.936 \pm 2.33 \cdot \sqrt{\frac{0.936 \cdot (1-0.936)}{250}} \right]$$

$$= [0.8999, 0.97206] \Rightarrow \text{length} \approx 0.07216$$

Using Wilson's estimate yields a slightly wider interval

3. A fast-food chain decided to carry out an experiment to assess the influence of advertising expenditure on sales. Different relative changes in advertising expenditure, compared to the previous year, were made in 8 regions of the country, and resulting changes in sales levels were observed. The data yield the following least squares regression line

$$\text{increase in sales} = 3.2958333 + 0.5391026 \text{ increase in advertising expenditure}$$

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	3.2958333	1.207511	2.73	0.0342
increase in advertising expenditure	0.5391026	0.153664	3.51	0.0127

↑ Slope b ↑ estimate b ↓ SE_b ↓ t_{obs}

- (a) Using the output given above, find a 90% confidence interval for the slope of the population regression line.

a 90% CI: $C = 90\% \Rightarrow (1 - \alpha) = .90 \Rightarrow \alpha = .10$
 $\Rightarrow \alpha/2 = 0.05 \Rightarrow t^* = 1.943, df = n - 2 = \underline{\underline{6}}$

$$b \pm 1.943 \cdot SE_b \Rightarrow [0.5391 \pm 1.943 \cdot 0.153664]$$

$$\Rightarrow [0.2405, 0.8376]$$

- (b) Based on an appropriate hypothesis test, can you conclude that increased advertising expenditures are positively related to increased sales? Carefully carry out all necessary steps for a hypothesis test and state your conclusions. $\alpha = 0.05$

• $H_0: \beta = 0$ vs. $H_a: \beta > 0$

(see above

output)

• $t_{obs} = \frac{b}{SE_b} = \frac{0.5391}{0.153664} = 3.51$

• p-value: $0.0127 \div 2$

$= 0.00635$ (because this is only a one-sided alternative $H_a: \beta > 0$)

\Rightarrow p-value $< \alpha = 0.05 \Rightarrow$ able to reject at the 0.05 level

\Rightarrow There is statistically significant evidence at the 0.05 level to conclude that increased advertising expenditures are positively related to increased sales?

Use z^* because σ is known

4. A college admissions officer for an M.B.A. program has determined that historically applicants have undergraduate grade point averages that are normally distributed with standard deviation of 0.45. From a random sample of 25 applications from the current year, the sample mean grade point average is 2.90.

(a) Find a 94% confidence interval for the population mean.

$$n = 25, \sigma = 0.45, \bar{x} = 2.90$$

$$C = 94 \Rightarrow (1 - \alpha) = .94 \Rightarrow \alpha = 0.06 \Rightarrow \alpha/2 = 0.03 \Rightarrow z^* = 1.88$$

$$\Rightarrow \left[\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \right] \Rightarrow \left[2.90 \pm 1.88 \cdot \frac{0.45}{\sqrt{25}} \right]$$

$$\Rightarrow [2.7308, 3.0692]$$

- (b) Based on the sample results, a statistician computes for the population mean a confidence interval extending from 2.81 to 2.99. Find the confidence level associated with this interval.

$$\text{length of CI} = 2.99 - 2.81 = 0.18$$

$$\Rightarrow \text{margin of error } m = z^* \frac{\sigma}{\sqrt{n}} = 0.09$$

$$\Rightarrow 0.09 = z^* \cdot \frac{0.45}{5} \Rightarrow z^* = 1 \Rightarrow \alpha/2 = 0.1587$$

$$\Rightarrow \alpha = 0.3174 \Rightarrow \approx 68\% \text{ CI}$$

5. Suppose Brent Matthews, manager of a Sam's Club in Tennessee wants to estimate the mean number of gallons of milk that are sold during a typical weekday. Brent checked the sales records for random sample of 16 days and found the mean number of gallons sold is 150 gallons per day, the sample standard deviation is 12 gallons. With 95% confidence estimate the number of gallons that Brent should stock daily.

$$n = 16, \bar{x} = 150, s = 12, C = 95 \Rightarrow (1 - \alpha) = .95 \Rightarrow$$

$$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$$

$$\Rightarrow t^* = 2.131, df = n - 1 = \underline{\underline{15}}$$

$$\Downarrow \left[\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}} \right]$$

$$= \left[150 \pm 2.131 \cdot \frac{12}{\sqrt{16}} \right] \Rightarrow [143.607, 156.393]$$

6. The student government association at a university wants to estimate the percentage of the student body that supports a change being considered in the academic calendar of the university for the next academic year. How many students should be surveyed if a 90% confidence interval is desired and the margin of error is only to be 2%?

as we don't know anything about p or \hat{p} , ~~we~~ we have to assume the most conservative p which is 0.5

↓

$$n \geq \left(\frac{z^*}{m} \right)^2 \cdot p(1-p)$$

$$= \left(\frac{1.645}{0.02} \right)^2 \cdot 0.5(1-0.5)$$

$$= 1691.26$$

⇓

1692 students

$$C = 90 \Rightarrow (1-\alpha) = .9$$

$$\Rightarrow \alpha = .1 \Rightarrow \alpha/2 = .05$$

$$\Rightarrow z^* = 1.645$$

$$m = 0.02$$