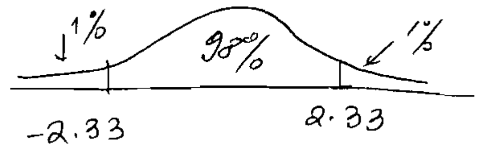
Worksheet for Confidence Intervals and Sample Size calculations

It is known that the standard deviation in the volumes of 24-ounce (710-mL) bottles of natural spring water bottled by a particular company is 6 mL. Ninety bottles are randomly sampled and measured and yielded a mean volume of 708 mL.

Find a 98% confidence interval for the true population mean to check if the bottling process still is on target.

$n = 90$, $\bar{X} = 708$ mL $\quad \sigma = 6 \quad C = 98\% \Rightarrow$



$\Downarrow$

$z^* = 2.33$

CI is $708 \pm 2.33 \times \frac{6}{\sqrt{90}} \Rightarrow (706.53,\ 709.47)$

Interpretation: We are 98% confident that the mean μ is between 706.53 and 709.47

This CI does not capture the target of 710 mL, so this is a good indication that bottling process is not on target.

The description of the problem did not make any statement about the distribution of the bottle volumes? Why or why not would this be a concern?

We didn't need to know the distribution of the original population, we used CLT with $n = 90 > 30$ to ensure that $\bar{X}$ was normally distributed.

The president of the Bona Fide Washing Machine Company (BFWMC) tells the firm's statistician that it is important he be able to estimate the mean longevity of motors received from supplier I with a 95% probability of an error no more than 20 hours. If he knows that the standard deviation of the lengths of life of motors received from supplier I is 400 hours, how large a sample must the firm take of the motors received from supplier I to be certain that this is true?

$\sigma = 400$; $C = 95\% \Rightarrow$ critical value $z^* = 1.96$

margin of error: $m = 20$.

Thus $n = \left(\frac{1.96 \times 400}{20}\right)^2 = 1536.64 \Rightarrow$ rounding up

$\Rightarrow$ we need to have a sample of size at least 1537 to ensure a 95% CI with a margin of error no greater than 20 hours.

The BFWMC statistician says that the 90% confidence interval for the mean length of life of motors received from supplier II is 4,500 to 4,800 hours, based on a sample of 36 motors. The statistician also says that the standard deviation of the lengths of life of motors received from supplier II is 500 hours. Is there any contradiction between these statements? If so, what is the contradiction?

$C = 90\%$; $n = 36$; $\sigma = 500$.

90% CI is $(4{,}500, 4800) \Rightarrow$ this interval has a width of 300.

If we calculate the width for $C = 90\%$, $n = 36$ and $\sigma = 500$, we have $z^* = 1.64 \Rightarrow m = \frac{1.64 \times 500}{\sqrt{36}} = 136.67 \Rightarrow$ width $= 273.33$

Since calculated width (273.33) is less than given width (300) we have a contradiction.

(Note here: you could have used $z^* = 1.65 \Rightarrow$ width $= 275$ or $z^* = 1.645 \Rightarrow$ width $= 274.17$] contradiction in either of these cases).