1 (Problem 47 - p. 121)

Decision variables:
x_1: no. Mon.-Tues. day-off pairs
x_2: no. Tues.-Wed. day-off pairs
x_3: no. Wed.-Thur. day-off pairs
x_4: no. Thur.-Fri. day-off pairs
x_5: no. Fri.-Sat. day-off pairs
x_6: no. Sat.-Sun. day-off pairs
x_7: no. Sun.-Mon. day-off pairs
u_1: no. Tues. nonconsecutively paired days off
u_2: no. Wed. nonconsecutively paired days off
u_3: no. Thur. nonconsecutively paired days off
u_4: no. Fri. nonconsecutively paired days off
u_5: no. Sat. nonconsecutively paired days off
u_6: no. Sun. nonconsecutively paired days off
u_7: no. Mon. nonconsecutively paired days off

LP model:

$$
\begin{array}{llll}
\min & z = \sum_{i=1}^{7} u_i & & \\
\text{s.t.} & x_1 + x_2 + u_1 & = 6 & \text{(\# officers whose day-off is on Tuesday)} \\
& x_2 + x_3 + u_2 & = 5 & \text{(\# officers whose day-off is on Wednesday)} \\
& x_3 + x_4 + u_3 & = 14 & \text{(\# officers whose day-off is on Thursday)} \\
& x_4 + x_5 + u_4 & = 9 & \text{(\# officers whose day-off is on Friday)} \\
& x_5 + x_6 + u_5 & = 2 & \text{(\# officers whose day-off is on Saturday)} \\
& x_6 + x_7 + u_6 & = 12 & \text{(\# officers whose day-off is on Sunday)} \\
& x_7 + x_1 + u_7 & = 12 & \text{(\# officers whose day-off is on Monday)} \\
& \sum_{i=1}^{7} x_i + \frac{1}{2}\sum_{i=1}^{7} u_i & = 30 & \text{(\# total police officers)} \\
& u_i \le \sum_{j \neq i} u_j & , i = 1, ..., 7 & \text{(nonconsective day-off on day } i \text{ should also appear on day } j \neq i) \\
& x_i, u_i & \ge 0,\ i = 1, ..., 7 & \text{(Sign constraints)}
\end{array}
$$

2 (Problem 48 - p. 121)

Notations:
i: Index of months. 1 - Januray, 2 - February, 3 - March, 4 - April.
c_i: Purchasing price of corn in month i (dollar per ton), $i = 1, ..., 4$
p_i: Selling price of corn in month i (dollar per ton), $i, ..., 4$

Decision variables:
x_i: Amount of corn bought at the first day of month i, $i = 1, ..., 4$
y_i: Amount of corn sold at the last day of month i, $i = 1, ..., 4$

LP model:

2 continued on next page. . .

$$\begin{array}{llll} \max & z = \sum_{i=1}^{4}(p_i y_i - c_i x_i) & & \\ \text{s.t.} & 50 + \sum_{j=1}^{i-1}(x_j - y_j) + x_i \le 100 & , i = 1, ..., 4 & \text{(Warehouse capacity)} \\ & y_i \le 50 + \sum_{j=1}^{i-1}(x_j - y_j) + x_i & , i = 1, ..., 4 & \text{(Available corn to be sold)} \\ & c_i x_i \le 1000 + \sum_{j=1}^{i-1}(p_j y_j - c_j x_j) & , i = 1, ..., 4 & \text{(Available corn to be bought)} \\ & x_i, y_i \ge 0 & , i = 1, ..., 4 & \text{(Sign constraints)} \end{array}$$

3 (Problem 51 - p. 122)

Let a grade 0 transistor represents a defective transistor.

Notations:
d_i: Number of monthly demand of grade i transistor, $i = 1, ..., 4$
u_{ij}: Proportion of yield of grade i transistor produced by method (%) j, $i = 0, ..., 4$, $j = 1, 2$
c_j: Dollars spent on producing one transistor by method j, $j = 1, 2$
v_{ik}: Proportion of yield of grade i transistor by refiring grade k transistor (%), $i = 0, ..., 4$, $k = 0, ..., 3$

Decision variables:
x_j: Number of transistors monthly produced by method j, $j = 1, 2$
y_k: Number of grade k transistors to be refired, $k = 0, ..., 4$

LP model:

$$\begin{array}{lll} \min & z = \sum_{j=1}^{2} c_j x_j + 25 \sum_{k=0}^{3} y_k & \\ \text{s.t.} & \sum_{j=1}^{2} u_{ij} x_j + \sum_{k=0}^{3} v_{ik} y_k - y_i & \ge d_i,\ i = 1, ..., 4 \quad \text{(Final amount of grade } i \text{ transistors)} \\ & y_i & \le \sum_{j=1}^{2} u_{ij} x_j,\ i = 0, ..., 4 \quad \text{(Available grade } i \text{ transistors to be refired)} \\ & \sum_{j=1}^{2} x_j + \sum_{k=0}^{3} y_k & \le 20,000 \quad \text{(Capacity of furnance)} \\ & x_j, y_k & \ge 0, ,\ j = 1, 2,\ k = 0, ..., 3 \quad \text{(Sign constraints)} \end{array}$$

4 (Problem 52 - p. 122)

Notations:
i: index of type of paper. 1 - box board, 2 - tissue paper, 3 - newsprint, 4 - book paper
k: index of method used to process inputs. 1 - de-inking, 2 - asphalt dispersion
c_i: Dollars needed to purchase a ton of paper i
p_i: Proportion of pulp in paper i (%)
d_j: Demand of grade j recycled paper (ton)
y_k: Yield of method k (%)

Decision variables:

4 continued on next page...

x_{ijk}: Tons of paper i used to produce grade j recycled paper by using method k, $i = 1, ..., 4$, $j = 1, 2, 3$, $k = 1, 2$

LP model:

$$\begin{array}{lll}
\min & z = 20 \sum_{i=1}^{4} \sum_{j=1}^{3} x_{ij1} + 15 \sum_{i=1}^{4} \sum_{j=1}^{3} x_{ij2} + \sum_{i=1}^{4} \sum_{j=1}^{3} \sum_{k=1}^{2} c_i x_{ijk} & \text{(Total cost)} \\
\text{s.t.} & \sum_{i=1}^{4} \sum_{j=1}^{3} x_{ijk} \leq 3,000, \qquad k = 1, 2 & \text{(Process capacity)} \\
& x_{11k}, x_{21k} = 0, \qquad k = 1, 2 & \text{(Input for grade 1)} \\
& x_{32k} = 0, \qquad k = 1, 2 & \text{(Input for grade 2)} \\
& x_{43k} = 0, \qquad k = 1, 2 & \text{(Input for grade 3)} \\
& \sum_{i=1}^{4} \sum_{k=1}^{2} y_k p_i x_{ijk} \geq d_j, \qquad j = 1, 2, 3 & \text{(Demand)} \\
& x_{ijk} \geq 0, \qquad i = 1, ..., 4, \ j = 1, 2, 3, \ k = 1, 2 & \text{(Sign constraints)}
\end{array}$$