1 (Problem 2 - p. 254)

LINGO model:

SETS:
types / 1 2 / : lbound, ruby, diamond, price, cost, x;
ENDSETS

DATA:
lbound = 11 0;
ruby = 2 1;
diamond = 4 1;
price = 10 6;
cost = 5 4;
tot_ruby = 30;
tot_diamond = 50;
ENDDATA

MAX = @SUM(types(i): (price(i)-cost(i))*x(i));

@SUM(types(i): ruby(i)*x(i)) <= tot_ruby;
@SUM(types(i): diamond(i)*x(i)) <= tot_diamond;
@FOR(types: x >= lbound;);

LINGO solution report:

Global optimal solution found.
Objective value: 67.00000
Infeasibilities: 0.000000
Total solver iterations: 1

Model Class: LP

Total variables: 2
Nonlinear variables: 0
Integer variables: 0

Total constraints: 5
Nonlinear constraints: 0

Total nonzeros: 8
Nonlinear nonzeros: 0


<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOT_RUBY</td>
<td>30.00000</td>
<td>0.000000</td>
</tr>
<tr>
<td>TOT_DIAMOND</td>
<td>50.00000</td>
<td>0.000000</td>
</tr>
<tr>
<td>LBOUND( 1)</td>
<td>11.00000</td>
<td>0.000000</td>
</tr>
<tr>
<td>LBOUND( 2)</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>RUBY( 1)</td>
<td>2.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>RUBY( 2)</td>
<td>1.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
### LINGO range report:

Ranges in which the basis is unchanged:

#### Objective Coefficient Ranges:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Current Coefficient</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(1)</td>
<td>5.000000</td>
<td>3.000000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>X(2)</td>
<td>2.000000</td>
<td>INFINITY</td>
<td>0.7500000</td>
</tr>
</tbody>
</table>

#### Righthand Side Ranges:

<table>
<thead>
<tr>
<th>Row</th>
<th>Current RHS</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>30.000000</td>
<td>INFINITY</td>
<td>2.000000</td>
</tr>
<tr>
<td>3</td>
<td>50.000000</td>
<td>2.000000</td>
<td>6.000000</td>
</tr>
<tr>
<td>4</td>
<td>11.000000</td>
<td>1.500000</td>
<td>1.000000</td>
</tr>
<tr>
<td>5</td>
<td>0.000000</td>
<td>6.000000</td>
<td>INFINITY</td>
</tr>
</tbody>
</table>

a) $67 - 2 \times (50 - 46) = 59$ (dollars)

Dual price of capacity of diamonds (Row 3) is 2. Also, RHS of row 3 is allowable to decrease by 6 without change of the the dual price.

b) $(x_1^*, x_2^*) = (11, 6)$

Because allowable decrease of coefficient of $x_2$ in objective function is $0.75 > (6 - 5.5)$, the optimal solution will not be changed.

c) $67 - 3 = 64$ (dollars)

RHS of row 4 is allowable to increase by 1.5. Therefore, when RHS of row 4 increases by 1, the dual price of row 4 does not change.

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2 continued on next page...
2 (Problem 11 - p. 259)

LINGO model:

SETS:
   PLANT/ OH, CA, TN/: Labor, Machine, Capacity, X;
ENDSETS

DATA:
   Labor = 2 1.5 1.1;
   Machine = 1 1.5 2.5;
   Capacity = 1000 900 2000;
   CLabor = 30;
   CMachine = 10;
   Demand = 1800;
ENDDATA

[OBJ] MIN = @SUM( PLANT(i): (CLabor*Labor(i)+CMachine*Machine(i))*X(i));

! capacity;
@FOR( PLANT(i):
   [CAPA] Machine(i)*X(i) <= Capacity(i);
);

! demand;
[TARGET] @SUM( PLANT(i): X(i)) >= Demand;

! sign;
@FOR( PLANT(i):
   [SIGN] X(i) >= 0;
);

LINGO solution report:

Global optimal solution found.
Objective value: 110400.0
Infeasibilities: 0.000000
Total solver iterations: 0

Model Class: LP

Total variables: 3
Nonlinear variables: 0
Integer variables: 0

Total constraints: 8
Nonlinear constraints: 0

Total nonzeros: 12
Nonlinear nonzeros: 0
Variable Value Reduced Cost
CLABOR 30.00000 0.000000
CMACHINE 10.00000 0.000000
DEMAND 1800.000 0.000000
LABOR (OH) 2.000000 0.000000
LABOR (CA) 1.500000 0.000000
LABOR (TN) 1.100000 0.000000
MACHINE (OH) 1.000000 0.000000
MACHINE (CA) 1.500000 0.000000
MACHINE (TN) 2.500000 0.000000
CAPACITY (OH) 1000.000 0.000000
CAPACITY (CA) 900.0000 0.000000
CAPACITY (TN) 2000.000 0.000000
X (OH) 400.0000 0.000000
X (CA) 600.0000 0.000000
X (TN) 800.0000 0.000000

Row Slack or Surplus Dual Price
OBJ 110400.0 -1.000000
CAPA (OH) 600.0000 0.000000
CAPA (CA) 0.000000 6.666667
CAPA (TN) 0.000000 4.800000
TARGET 0.000000 -70.00000
SIGN (OH) 400.0000 0.000000
SIGN (CA) 600.0000 0.000000
SIGN (TN) 800.0000 0.000000

LINGO range report:

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Current Coefficient</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>X (OH)</td>
<td>70.00000</td>
<td>INFINITY</td>
<td>10.00000</td>
</tr>
<tr>
<td>X (CA)</td>
<td>60.00000</td>
<td>10.00000</td>
<td>INFINITY</td>
</tr>
<tr>
<td>X (TN)</td>
<td>58.00000</td>
<td>12.00000</td>
<td>INFINITY</td>
</tr>
</tbody>
</table>

Righthand Side Ranges:

<table>
<thead>
<tr>
<th>Row</th>
<th>Current RHS</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPA (OH)</td>
<td>1000.000</td>
<td>INFINITY</td>
<td>600.0000</td>
</tr>
<tr>
<td>CAPA (CA)</td>
<td>900.0000</td>
<td>600.0000</td>
<td>900.0000</td>
</tr>
<tr>
<td>CAPA (TN)</td>
<td>2000.000</td>
<td>1000.0000</td>
<td>1500.0000</td>
</tr>
<tr>
<td>TARGET</td>
<td>1800.000</td>
<td>600.0000</td>
<td>400.0000</td>
</tr>
</tbody>
</table>
a) Optimal solution: \((X(OH)^*, X(CA)^*, X(TN)^*) = (400, 600, 800)\)
400 automobiles at plant OH, 600 at CA, 800 at TN

b) $5
Allowable decrease of coefficient of \(X(OH)\) in objective function is 10. It means that the optimal solution will not be changed until the wage rate (hours/automobile) times the labor rate (hours/automobile) at plant OH is decreased by 10. Because the labor rate of plant OH is 2, the minimum decrease of the wage rate to increase \(X(OH)^*\) is $5.

c) $70 / Yes
Dual price of the demand constraint (TARGET) is -70, and allowable increase of RHS of TARGET is 600. Because RHS of TARGET is allowable to decrease by 400, the dual price will be changed if RHS of the demand constraint is 1000.

d) $9,000
If the labor rate of plant CA is changed from 1.5 to 1, the coefficient of \(X(CA)\) in objective function is changed from 60 \((= 30 \times 1.5 + 10 \times 1.5)\) to 45 \((= 30 \times 1 + 10 \times 1.5)\). Because the coefficient is allowable to decrease infinitely without change of the optimal solution, the cost reduction is \((60 - 45) \times X(CA)^* = 9000\).

e) $14,000
Because the allowable increase of RHS of the demand constraint (TARGET) is 600, we can increase RHS of TARGET without change of the dual price of TARGET. Therefore, increase of costs is \(200 \times 70 = 14,000\).

f) \((X(OH)^*, X(CA)^*, X(TN)^*) = (400, 600, 800)\)
If labor costs in plant CA is increased by $2, the coefficient of \(X(CA)\) in objective function is changed from 60 \((= 30 \times 1.5 + 10 \times 1.5)\) to 63 \((= 32 \times 1.5 + 10 \times 1.5)\). Because the coefficient is allowable to be increased by 10 without any change of the optimal solution, increase of labor costs from $30 to $32 does not change the optimal solution. However, the objective value will be changed from 110,400 to 112,200.

3 (Problem 12 - p. 259)
LINGO model:

SETS:
    Product/ 1 2 3 4 /: Machine1, Machine2, Skilled, Unskilled, Price, X;
ENDSETS

DATA:
    Machine1 = 11 7 6 5;
    Machine2 = 4 6 5 4;
    Skilled = 8 5 4 6;
    Unskilled = 7 8 7 4;
    Price = 300 260 220 180;
    CapaMach1 = 700;
    CapaMach2 = 500;
CapaSkill = 600;
CapaUnskill = 650;
CostSkill = 8;
CostUnskill = 6;
ENDDATA

[OBJ] MAX = @SUM(Product(i): Price(i)*X(i)) - @SUM(Product(i): (CostSkill*Skilled(i)+CostUnskill)*X(i))

! capacity;
[AVAILABLE_MACHINE1] @SUM(Product(i): Machine1(i) * X(i)) <= CapaMach1;
[AVAILABLE_MACHINE2] @SUM(Product(i): Machine2(i) * X(i)) <= CapaMach2;
[AVAILABLE_SKILLED] @SUM(Product(i): Skilled(i) * X(i)) <= CapaSkill;
[AVAILABLE_UNSKILLED] @SUM(Product(i): Unskilled(i) * X(i)) <= CapaUnskill;

! sign;
@FOR(Product(i):
  [SIGN] X(i) >= 0;
); 

LINGO solution report:

Global optimal solution found.
Objective value: 15433.33
Infeasibilities: 0.000000
Total solver iterations: 4
Model Class: LP

Total variables: 4
Nonlinear variables: 0
Integer variables: 0

Total constraints: 9
Nonlinear constraints: 0

Total nonzeros: 24
Nonlinear nonzeros: 0

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPAMACH1</td>
<td>700.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>CAPAMACH2</td>
<td>500.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>CAPASKILL</td>
<td>600.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>CAPAUNSKILL</td>
<td>650.0000</td>
<td>0.000000</td>
</tr>
<tr>
<td>COSTSKILL</td>
<td>8.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>COSTUNSKILL</td>
<td>6.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>MACHINE1( 1)</td>
<td>11.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>MACHINE1( 2)</td>
<td>7.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>MACHINE1( 3)</td>
<td>6.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>MACHINE1( 4)</td>
<td>5.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
MACHINE2( 1) 4.000000 0.000000
MACHINE2( 2) 6.000000 0.000000
MACHINE2( 3) 5.000000 0.000000
MACHINE2( 4) 4.000000 0.000000
SKILLED( 1) 8.000000 0.000000
SKILLED( 2) 5.000000 0.000000
SKILLED( 3) 4.000000 0.000000
SKILLED( 4) 6.000000 0.000000
UNSKILLED( 1) 7.000000 0.000000
UNSKILLED( 2) 8.000000 0.000000
UNSKILLED( 3) 7.000000 0.000000
UNSKILLED( 4) 4.000000 0.000000
PRICE( 1) 300.0000 0.000000
PRICE( 2) 260.0000 0.000000
PRICE( 3) 220.0000 0.000000
PRICE( 4) 180.0000 0.000000
X( 1) 16.66667 0.000000
X( 2) 50.00000 0.000000
X( 3) 0.000000 1.629630
X( 4) 33.33333 0.000000

<table>
<thead>
<tr>
<th>Row</th>
<th>Slack or Surplus</th>
<th>Dual Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJ</td>
<td>15433.33</td>
<td>1.000000</td>
</tr>
<tr>
<td>AVAILABLE_MACHINE1</td>
<td>0.000000</td>
<td>10.51852</td>
</tr>
<tr>
<td>AVAILABLE_MACHINE2</td>
<td>0.000000</td>
<td>6.222222</td>
</tr>
<tr>
<td>AVAILABLE_SKILLED</td>
<td>16.66667</td>
<td>0.000000</td>
</tr>
<tr>
<td>AVAILABLE_UNSKILLED</td>
<td>0.000000</td>
<td>7.629630</td>
</tr>
<tr>
<td>SIGN( 1)</td>
<td>16.66667</td>
<td>0.000000</td>
</tr>
<tr>
<td>SIGN( 2)</td>
<td>50.00000</td>
<td>0.000000</td>
</tr>
<tr>
<td>SIGN( 3)</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>SIGN( 4)</td>
<td>33.33333</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

LINGO range report:

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Current Coefficient</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>X( 1)</td>
<td>194.0000</td>
<td>28.00000</td>
<td>44.00000</td>
</tr>
<tr>
<td>X( 2)</td>
<td>172.0000</td>
<td>37.33333</td>
<td>1.725490</td>
</tr>
<tr>
<td>X( 3)</td>
<td>146.0000</td>
<td>1.629630</td>
<td>INFINITY</td>
</tr>
<tr>
<td>X( 4)</td>
<td>108.0000</td>
<td>8.000000</td>
<td>8.615385</td>
</tr>
</tbody>
</table>

Righthand Side Ranges:
a) $1.63
In the above solution report, reduced of $X_3$ is 1.63. It means that 1) $X_3$ is a non-basic variable in the current optimal solution and 2) it will be a basic variable if the coefficient of $X_3$ in the objective function is improved by 1.63. Because this is a maximization problem, improving direction of the objective function is increasing direction. Therefore, if the sales price of product 3 increases by $1.63, the optimal solution will contain positive $X_3^*$. 

b) $(X_1^*, X_2^*, X_3^*, X_4^*) = (16.67, 50.00, 0.00, 33.33)$
Because allowable decrease of the coefficient of $X_1$ is 44, the optimal solution will not be changed if selling price of product 1 is decreased by only $10. However, the objective value will be changed from 15,433 to 15,266($=15,433 - 10 \times 16.67$).

c) Allowable Increases for both machines (AVAILABLE_MACHINE1 and AVAILABLE_MACHINE2) are 14.06 and 9.68, respectively and the corresponding dual prices are 10.52 and 6.22, respectively. Therefore, the company would pay some money (at most the dual prices) for one additional hour for each machine.

d) Dual price of skilled labor hours (AVAILABLE_SKILLED) is 0. On the other hand, dual price of unskilled labor hours (AVAILABLE_UNSKILLED) is 7.63. Also, allowable increase of unskilled labor hours is 47.37. It means that additional one hour of unskilled labor will increase the total profit by $7.63 whereas additional skilled labor does not affect the profit. Therefore, only additional unskilled labor is valuable and the company would pay at most $7.63 for an extra hour of unskilled labor.

e) $15,433
Because the dual price of skilled labor hours is 0 and the allowable increase of skilled labor hours is infinite, the objective value will not be changed.