1 (Problem 1 - p. 71)

Decision variables:
x_i: Tons of factory i waste to be processed, $i = 1, 2, 3$

LP model:

$$\begin{array}{lll} \min & z = 15x_1 + 10x_2 + 20x_3 & \text{(Total cost)} \\ \text{s.t.} & 0.10x_1 + 0.20x_2 + 0.40x_3 \leq 30 & \text{(Amount of pollutant 1 to be reduced)} \\ & 0.45x_1 + 0.25x_2 + 0.30x_3 \leq 40 & \text{(Amount of pollutant 2 to be reduced)} \\ & x_1, x_2, x_3 \geq 0 & \text{(Sign constraints)} \end{array}$$

For LP assumptions, there are many possible answers.
(Example) Proportionality assumption may be satisfied because the amount of waste and the amount of resource to process waste is linearly dependent and because the amount of waste processed in one factory do not affect processing costs in other factories.
Additivity assumption may be satisfied because reduction rates of pollutant by processing waste are constant and because reduction of pollutants in one factory to not affect reduction in other factories.
Divisibility assumption is satisfied because the weight of waste can be 0.5 ton which means 500 kilograms.
Certainty is satisfied because the unit costs and the reduction rate may have been observed.

2 (Problem 2 - p. 71)

Decision variables:
x_i: Number of valves to be ordered to supplier i in each month, $i = 1, 2, 3$

LP model:

$$\begin{array}{lll} \min & z = 5x_1 + 4x_2 + 3x_3 & \text{(Total cost)} \\ \text{s.t.} & 0.40x_1 + 0.30x_2 + 0.20x_3 \geq 500 & \text{(Amount of large valves to be purchased)} \\ & 0.40x_1 + 0.35x_2 + 0.20x_3 \geq 300 & \text{(Amount of medium valves to be purchased)} \\ & 0.20x_1 + 0.35x_2 + 0.60x_3 \geq 300 & \text{(Amount of small valves to be purchased)} \\ & x_i \leq 700, i = 1, 2, 3 & \text{(Capacity of each supplier)} \\ & x_i \geq 0, i = 1, 2, 3 & \text{(Sign constraints)} \end{array}$$

3 (Problem 3 - p. 92)

Decision variables:
x_1: Dollars to be invested in bonds
x_2: Dollars to be invested in home loans
x_3: Dollars to be invested in auto loans
x_4: Dollars to be invested in personal loans

LP model:

3 continued on next page...

$$
\begin{array}{lll}
\max & z = 0.10x_1 + 0.16x_2 + 0.13x_3 + 0.20x_4 & \text{(Annual return on investments)} \\
\text{s.t.} & x_1 - x_4 & \geq 0 \quad \text{(restriction a)} \\
& -x_2 + x_3 & \geq 0 \quad \text{(restriction b)} \\
& 0.25x_1 + 0.25x_2 + 0.25x_3 - 0.75x_4 & \geq 0 \quad \text{(restriction c)} \\
& x_1 + x_2 + x_3 + x_4 & \leq 500,000 \quad \text{(Available budget)} \\
& x_i & \geq 0, i = 1,2,3,4 \quad \text{(Sign constraints)}
\end{array}
$$

4 (Problem 7 - p. 93)

Decision variables:
x_{ij}: Oz of chemical i used to produce drug j

Revenue (in dollar): $6(x_{11} + x_{21}) + 5(x_{12} + x_{22})$
Cost (in dollar): $6(x_{11} + x_{12}) + 4(x_{21} + x_{22})$
Profit (in dollar): $2x_{21} - x_{12} + x_{22}$

LP model:

$$
\begin{array}{lll}
\max & z = 2x_{21} - x_{12} + x_{22} & \text{(Total profit)} \\
\text{s.t.} & 0.3x_{11} - 0.7x_{21} & \geq 0 \quad \text{(at least 70\% chemical 1 in drug 1)} \\
& -0.6x_{12} + 0.4x_{22} & \geq 0 \quad \text{(at least 60\% chemical 2 in drug 2)} \\
& x_{11} + x_{21} & \leq 40 \quad \text{(Amount of drug 1 to be sold)} \\
& x_{12} + x_{22} & \leq 30 \quad \text{(Amount of drug 2 to be sold)} \\
& x_{11} + x_{12} & \leq 45 \quad \text{(Amount of chemical 1 to be purchased)} \\
& x_{21} + x_{22} & \leq 40 \quad \text{(Amount of chemical 2 to be purchased)} \\
& x_{ij} & \geq 0, \; i = 1,2, \; j = 1,2 \quad \text{(Sign constraints)}
\end{array}
$$

5 (Problem 14 - p. 94)

Notations:
Index of input (i) – 1: reformate / 2: FCG / 3: ISO / 4: POL / 5: MTB / 6: BUT
Index of product (j) – 1: regular gasoline / 2: premium gasoline
Index of attribute of inputs (k) – 1: RON / 2: RVP / 3: ASTM70 / 4: ASTM130
c_i: Daily available liters of input i, $i = 1, ..., 5$
a_{ik}: Attribute k of input i, $i = 1, ..., 6$, $k = 1, ..., 4$
d_j: Daily demanded liters of product j, $j = 1, 2$
r_{jk}: Required attribute k of product j, $j = 1, 2$, $k = 1, ..., 4$
p_j: Sale price of product j per liter, $j = 1, 2$

Decision Variables:
x_{ij}: Oz of chemical i used to produce drug j

LP model:

$$
\begin{aligned}
\max \quad & z = \sum_{i=1}^{6}\sum_{j=1}^{2}(p_j - 12.75)x_{ij} && \text{(Daily profit)}\\
\text{s.t.} \quad & \sum_{j=1}^{2} x_{ij} \le c_i,\ i = 1,...,5 && \text{(Availability of inputs)}\\
& \sum_{i=1}^{6}(a_{ik} - r_{jk})x_{ij} \ge 0,\ j = 1,2,\ k = 1,3,4 && \text{(RON and ASTM of gasoline)}\\
& \sum_{i=1}^{6}(a_{ik} - r_{jk})x_{ij} = 0,\ j = 1,2,\ k = 2 && \text{(RVP of gasoline)}\\
& \sum_{i=1}^{6} x_{ij} \ge d_j,\ j = 1,2 && \text{(Amount of gasoline to be produced)}\\
& 0.38x_{1j} - 0.62x_{2j} + 0.38x_{3j} + 0.38x_{4j} + 0.38x_{5j} + 0.38x_{6j} \ge 0,\ j = 1,2 && \text{(FCG limits)}\\
& x_{ij} \ge 0,\ i = 1,...,6,\ j = 1,2 && \text{(Sign constraints)}
\end{aligned}
$$

6 (Problem 2 - p. 97)

Decision Variables:
x_1: Number of unfinished tables to be manufactured
x_2: Number of unfinished chairs to be manufactured
x_3: Number of finished tables to be manufactured
x_4: Number of finished chairs to be manufactured

Revenue(in dollar): $70(x_1 - x_3) + 60(x_2 - x_4) + 140x_3 + 110x_4$
Cost(in dollar): $40x_1 + 30x_2$
Profit(in dollar): $(70 - 40)x_1 + (60 - 30)x_2 + (140 - 70)x_3 + (110 - 60)x_4$
Hours of labor(in hour): $2x_1 + 2x_2 + 3x_3 + 2x_4$

LP model:

$$
\begin{aligned}
\max \quad & z = 30x_1 + 30x_2 + 70x_3 + 50x_4 && && \text{(Profit)}\\
\text{s.t.} \quad & 40x_1 + 30x_2 && \le 40,000 && \text{(Amount of wood to be purchased)}\\
& 2x_1 + 2x_2 + 3x_3 + 2x_4 && \le 6,000 && \text{(Total hours of skilled labor)}\\
& x_1 - x_3 && \ge 0 && \text{(Finished tables are made by processing unfinished tables)}\\
& x_2 - x_4 && \ge 0 && \text{(Finished chairs are made by processing unfinished chairs)}\\
& x_i && \ge 0,\ i = 1,2,3,4 && \text{(Sign constraints)}
\end{aligned}
$$

7 (Problem 7 - p. 98)

Notations:
Index of milk (i) – 1: high-fat milk / 2: low-fat milk
Index of cheese (j) – 1: cream cheese / 2: cottage cheese

Decision variables:
x_{ij}: Pounds of milk i to be directly used to produce cheese j (in lb/day)
y_i: Pounds of milk i to be evaporated to produce cream (in lb/day)

7 continued on next page...

z_j: Pounds of cream to produce cheese j (in lb/day)

Pounds of cheese j to be produced (in lb/day): $z_j + \sum_{i=1}^{2} x_{ij}$
Fat of cheese j (%): $(60x_{1j} + 30x_{2j})/(x_{1j} + x_{2j})$

Revenue(in dollar): $1.50(z_1 + \sum_{i=1}^{2} x_{i1}) + 0.90 \times 1.20(z_2 + \sum_{i=1}^{2} x_{i2})$
Purchasing Cost(in dollar): $0.80(y_1 + \sum_{j=1}^{2} x_{1j}) + 0.40(y_2 + \sum_{j=1}^{2} x_{2j})$
Production Cost(in dollar): $0.40 \sum_{i=1}^{2} y_i + 0.40 \sum_{j=1}^{2} \left(z_j + \sum_{i=1}^{2} x_{ij}\right)$

LP model:

$$\begin{aligned}
\max \quad & z = 0.30x_{11} - 0.12x_{12} + 0.70x_{21} + 0.28x_{22} - 1.20y_1 - 0.80y_2 + 1.10z_1 + 0.68z_2 \qquad \text{(Profit)} \\
\text{s.t.} \quad & z_1 + \textstyle\sum_{i=1}^{2} x_{i1} \geq 1000, \qquad \text{(Amount of cream cheese to be produced)} \\
& 1000 \leq 0.90(z_2 + \textstyle\sum_{i=1}^{2} x_{i2}) \geq 1000, \qquad \text{(Amount of cottage cheese to be produced)} \\
& z_1 + \textstyle\sum_{i=1}^{2} x_{i1} \leq 1500 \qquad \text{(Amount of cream cheese to be sold)} \\
& 0.90(z_2 + \textstyle\sum_{i=1}^{2} x_{i2}) \leq 2000 \qquad \text{(Amount of cottage cheese to be sold)} \\
& \textstyle\sum_{i=1}^{2} y_i \leq 2000 \qquad \text{(Capacity of evaporator)} \\
& \textstyle\sum_{j=1}^{2} \left(z_j + \sum_{i=1}^{2} x_{ij}\right) \leq 3000 \qquad \text{(Capacity of cheese machine)} \\
& 10x_{11} - 20x_{21} \geq 0 \qquad \text{(Fat of cream cheese)} \\
& 25x_{12} - 5x_{22} \geq 0 \qquad \text{(Fat of cottage cheese)} \\
& -0.40x_{11} - 0.40x_{21} + 0.60z_1 \geq 0 \qquad \text{(Proportion of cream in cream cheese)} \\
& -0.20x_{12} - 0.20x_{22} + 0.80z_2 \geq 0 \qquad \text{(Proportion of cream in cottage cheese)} \\
& 0.60y_1 + 0.30y_2 - z_1 - z_2 = 0 \qquad \text{(Yield of cream)} \\
& x_{ij}, y_i, z_j \geq 0, \ i = 1, 2, \ j = 1, 2 \qquad \text{(Sign constraints)}
\end{aligned}$$