IE312 F09 Hw2 Assignment

Due on Friday, October 23rd

Chapter 5 Review problem 1(p254), problem 3 (p. 254), problem 11 (p. 259), problem 12 (p. 259).

Note that for problems 11 & 12, I would like you to use LINGO rather than LINDO

Chapter 5 Review Problem 1(p254)

a) 4,000 hours labor change to 3,000 hours, original obj = 1,360,000

Row (7) Allowable increase = 300 Allowable decrease = 2400. This means as long as the labor availability is between 1600 and 4300 the current basis still optimal.

Row (7) dual price is 313.3333.

$OBJ_{new} = OBJ_{old} + \Delta labor$* dual price = 1,360,000 + (-1000)* 313.3333 = 1,046,667

b) Capacity of new York increases from 800 to 850 at cost of $5,000.
Row (3) Allowable increase = 100 Allowable decrease = 150. So under the new capacity (850) the current basis is still optimal. Without consider the $5,000 cost. We see how much more profit they can earn with 850 capacity. Dual price of row (3) is 133.3333
Δ profit = 50*133.3333 = 6,666.67
This is more than the cost, so they should hire the contractor,
$OBJ_{new} = OBJ_{old} + \Delta$ profit - cost = 1,360,000 + 6,666.67 – 5,000 = 1,361,667

c) If there is only 1 VAX in Los Angeles was produce, then the total profit will decrease by $33.333337. So this is the profit for a VAX produced in Los Angeles have to increase before Hal would want to produce VAXs in Los Angeles.

d) Since the dual price of labor hour is $313.3333, this is the profit increased by HAL if add one labor hour. Otherwise, they will decrease their profit. Total $313.3333.

Chapter 5 Review Problem 3(p254)

a) RM reduce from 90lb to 87lb, original obj = 274

Section *RightHand Side Ranges* Row (4) Allowable increase = 10 Allowable decrease = 23.34. This means as long as the RM availability is between 66.66 and 100, the current basis is still optimal.

Row (4) dual price is 2.6.

$OBJ_{new} = OBJ_{old} + \Delta RM$* dual price = 274 + (-3)* 2.6 = 266.2

b) The price of product 2 from $40 to $39.5, decreasing $0.5. Coefficient of P2 decreases 0.5*0.33 = 0.165.
Section *Obj Coefficient Ranges* Row (P2)Allowable decrease = 0.2, So under 0.165 short, the current basis is still optimal. $OBJ_{new} = OBJ_{old} + \Delta Coef.$* P2 = 274 + (-0.165)* 20 = 270.7

c) If 1lb of RM was purchased, from (a) we know that Allowable increase = 10. So the current basis is still optimal. The dual price is 2.6.
$OBJ_{new} = OBJ_{old} + \Delta RM*$ dual price = 274 + (1)* 2.6 = 276.6. The profit will increase by \$2.6. So this is the most that Wivco should pay for another pound of raw material.

d) If the labor hours increase from 200 to 201, then check Section *Obj Coefficient Ranges* Row (3) Allowable increase = 70, So on the condition of 201 labor hours, the current basis is still optimal. Row (3) dual price is 0.2.

$OBJ_{new} = OBJ_{old} + \Delta labor*$ dual price = 274 + (1)* 0.20 = 274.2
The profit will increase by \$0.2. So this is the most that Wivco should pay for additional labor hour.

Chapter 5 Review Problem 11 (p259)

Define the decision variables:

x_i: # of the automobile produced in Plant *i* daily. *i* = 1, 2, 3.

L_i: # of labor hours need in Plant *i* daily. *i* = 1, 2, 3.

M_i: # of machine time need in Plant *i* daily. *i* = 1, 2, 3.

Min $\$30L_1 + \$30L_2 + \$30L_3 + \$10M_1 + \$10M_2 + \$10M_3$

st.	$M_1 \le 1{,}000$	(machine capacity in Plant 1)
	$M_2 \le 900$	(machine capacity in Plant 2)
	$M_3 \le 2{,}000$	(machine capacity in Plant 3)
	$2x_1 \le L_1$	(labor needed in Plant 1)
	$1.5x_2 \le L_2$	(labor needed in Plant 2)
	$1.1x_3 \le L_3$	(labor needed in Plant 3)
	$x_1 \le M_1$	(machine time needed in Plant 1)
	$1.5x_2 \le M_2$	(machine time needed in Plant 2)
	$2.5x_3 \le M_3$	(machine time needed in Plant 3)
	$x_1 + x_2 + x_3 \ge 1800$	(target)
	$x_i, M_i, L_i \ge 0$ *i* = 1, 2, 3	(sign constraints)

a)

LINGO CODE:

```
MODEL:
 TITLE   Problem11;
 SETS:
   !Each Plant has an availability machine time and
```

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    labor hour and machine time need for one auto;

   PLANT/ OH, CA, TN/: Labor, Machine, MCapacity, NoAuto, NoLabor,NoMach;
   COST/LaborTime, MachTime/: PRICE;

 ENDSETS

 DATA:
 !Machine time availability;
  MCapacity = 1000, 900, 2000;

 !LABOR HOUR;
  Labor = 2, 1.5, 1.1;

 !Machine time;
  Machine = 1, 1.5, 2.5;

 !LABOR AND MACHINE TIME prices;
  PRICE = 30,  10;

 !Total production limit;
 Goal=1800;

 ENDDATA

 !Subject to machine capacity;

  @FOR( PLANT( R):
   [MCAP] NoMach( R) <= MCapacity( R);
  );

   !Labor Need;
  @FOR( PLANT( R):
   [LaborLmt] Labor( R)*NoAuto( R) <= NoLabor( R);
  );


   !Machine Need;
  @FOR( PLANT( R):
   [MachineLmt] Machine( R)*NoAuto( R) <= NoMach( R);
  );


   [Target]
    @SUM( PLANT( R): NoAuto( R))>=Goal;

  ! We want to minimize the total cost;
  [OBJECTIVE] MIN =
     @SUM( PLANT( R): NoLabor( R))*PRICE ( 1) + @SUM( PLANT( R): NoMach(
R))*PRICE ( 2);

END
```

OUTPUT FROM LINGO:

```
   Global optimal solution found.
```

```
Objective value:                                 110400.0
Total solver iterations:                                0


Model Title: Problem11

                     Variable           Value        Reduced Cost
                         GOAL        1800.000            0.000000
                  LABOR( OH)         2.000000            0.000000
                  LABOR( CA)         1.500000            0.000000
                  LABOR( TN)         1.100000            0.000000
                MACHINE( OH)         1.000000            0.000000
                MACHINE( CA)         1.500000            0.000000
                MACHINE( TN)         2.500000            0.000000
              MCAPACITY( OH)         1000.000            0.000000
              MCAPACITY( CA)         900.0000            0.000000
              MCAPACITY( TN)         2000.000            0.000000
                 NOAUTO( OH)         400.0000            0.000000
                 NOAUTO( CA)         600.0000            0.000000
                 NOAUTO( TN)         800.0000            0.000000
                NOLABOR( OH)         800.0000            0.000000
                NOLABOR( CA)         900.0000            0.000000
                NOLABOR( TN)         880.0000            0.000000
                 NOMACH( OH)         400.0000            0.000000
                 NOMACH( CA)         900.0000            0.000000
                 NOMACH( TN)         2000.000            0.000000
           PRICE( LABORTIME)         30.00000            0.000000
            PRICE( MACHTIME)         10.00000            0.000000

                          Row    Slack or Surplus      Dual Price
                   MCAP( OH)         600.0000            0.000000
                   MCAP( CA)         0.000000            6.666667
                   MCAP( TN)         0.000000            4.800000
               LABORLMT( OH)         0.000000            30.00000
               LABORLMT( CA)         0.000000            30.00000
               LABORLMT( TN)         0.000000            30.00000
             MACHINELMT( OH)         0.000000            10.00000
             MACHINELMT( CA)         0.000000            16.66667
             MACHINELMT( TN)         0.000000            14.80000
                       TARGET        0.000000           -70.00000
                    OBJECTIVE        110400.0           -1.000000
```

```
Ranges in which the basis is unchanged:
                      Objective Coefficient Ranges
                            Current        Allowable        Allowable
              Variable    Coefficient       Increase         Decrease
           NOAUTO( OH)            0.0       INFINITY         10.00000
           NOAUTO( CA)            0.0       10.00000         INFINITY
           NOAUTO( TN)            0.0       12.00000         INFINITY
          NOLABOR( OH)       30.00000       INFINITY         5.000000
          NOLABOR( CA)       30.00000       6.666667         30.00000
          NOLABOR( TN)       30.00000       10.90909         30.00000
           NOMACH( OH)       10.00000       INFINITY         10.00000
           NOMACH( CA)       10.00000       6.666667         INFINITY
           NOMACH( TN)       10.00000       4.800000         INFINITY
```

Righthand Side Ranges

Row	Current RHS	Allowable Increase	Allowable Decrease
MCAP(CA)	900.0000	600.0000	900.0000
MCAP(TN)	2000.000	1000.000	1500.000
LABORLMT(OH)	0.0	800.0000	INFINITY
LABORLMT(CA)	0.0	900.0000	INFINITY
LABORLMT(TN)	0.0	880.0000	INFINITY
MACHINELMT(OH)	0.0	400.0000	600.0000
MACHINELMT(CA)	0.0	600.0000	900.0000
MACHINELMT(TN)	0.0	1000.000	1500.000
TARGET	1800.000	600.0000	400.0000
MCAP(OH)	1000.000	INFINITY	600.0000

b) the smallest decrease is $5. Look at the coefficient of L1 as below.

Objective Coefficient Ranges

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
NOLABOR(OH)	30.00000	INFINITY	5.000000

c) Since the Row (target) ALLOWABLE INCREASE = 600, so add extra 1 automobile will not affect the current optimal basis, the dual price is $70. It will cause $70 to produce an additional automobile.

If the target is 1,000 automobiles, then it is beyond the ALLOWABLE DECREASE. So the current basis is changed, the result should be different from previous one.

d) The labor hours for 1 automobile from 1.5 hour to 1 hour. It corresponds to the labor cost decrease. At first, produce one automobile, 1.5 hours, $45. Now we only need 1 hour, $30 for one automobile. Assume in the same scenario, which means also need 1.5 hours, x is the money company would paid for one hour labor.

We have $\frac{45}{30} = \frac{30}{x} = \frac{\text{\$ for 1 car}}{\text{\$ for 1 labor hour}}$

x = 20

Check the coefficient of L2 as below. 20 is the range of keeping optimal basis.

	COEF	INCREASE	DECREASE
NOLABOR(CA)	30.000000	6.666667	30.000000

$OBJ_{new} = OBJ_{old} + \Delta price * L2 = 110,400 + (-10) * 900 = 101,400$

e) Since Row (target) ALLOWABLE INCREASE = 600, so add 200 more automobile is still in the current optimal basis. The dual price is $70. So we will increase cost by 70*200 = 14,000. The total cost = original cost + increased cost = 110,400 +14,000 = 124,400

f) The coefficient of NOLABOR(CA) increase by $2. There are in the range of ALLOWABLE INCREASE, so current optimal basis is not change. The new solution is the same: NOLABOR(OH) =800, NOLABOR(CA)=900, NOLABOR(TN)=880, NOMACH(OH)=400, NOMACH(CA) =900, NOMACH(CA) =2000, NOAUTO(OH)=400, NOAUTO(CA)=600, NOAUTO(TN)=800. The objective value change to original obj + $2*L2 = 110,400 + 2*900 = 112,200.

Chapter 5 Review Problem 12 (p259)

Define the decision variables:

x_i: # of product *i*. *i* = 1, 2, 3, 4.

L_j: # of Type *j* labor hours need. *j* = {skilled, unskilled}.

M_k: # of machine time *k* need. *k* =1, 2.

$$\text{Max} \sum_{i=1}^{4} sale(i) * x_i \quad - 8 * L_1 - 6 * L_2$$

st.

$11x_1 + 7x_2 + 6x_3 + 5x_4 \le M_1$	(machine time 1 needed)
$4x_1 + 6x_2 + 5x_3 + 4x_4 \le M_2$	(machine time 2 needed)
$M_1 \le 700$	(machine 1 capacity)
$M_2 \le 500$	(machine 2 capacity)
$8x_1 + 5x_2 + 4x_3 + 6x_4 \le L_1$	(machine time 1 needed)
$7x_1 + 8x_2 + 7x_3 + 4x_4 \le L_2$	(machine time 2 needed)
$L_1 \le 600$	(machine 1 capacity)
$L_2 \le 650$	(machine 2 capacity)
$x_i, M_j, L_k \ge 0 \quad i = 1, 2, 3, 4; j = 1, 2; k = 1, 2$	(sign constraints)

LINGO CODE:

```
MODEL:
 TITLE   Problem12;
 SETS:
   !Each Product has an availability machine time and
    labor hour and machine time need for one auto;

   Product/ One, Two, THR, Four/: NoProduct, PRICE;
   LaborHour/Labor1, labor2/:NoLabor, LCapacity, Lcost;
   MachineTime/M1, M2/:NoMachine, MCapacity;
   LCROSS(LaborHour, Product): LabP;
   MCROSS(MachineTime, Product): MachP;

 ENDSETS

 DATA:
 !Product prices;
  PRICE = 300, 260,220,180;
```

```
!Machine time availability;
 MCapacity = 700, 500;

!LABOR Cost;
 Lcost = 8, 6;

!Labor availability;
 LCapacity = 600, 650;

!LABOR HOUR;
 LabP = 8,5,4,6
        7,8,7,4;

!Machine time;
 MachP = 11,7, 6, 5
         4, 6, 5, 4;

ENDDATA

!Machine Need;
 @FOR( MachineTime(K):
  [MachineLmt] @SUM( PRODUCT(I):NoProduct(I)*MachP(K,I))<=NoMachine(K);
 );
!Subject to machine capacity;

 @FOR( MachineTime(K):
  [MachineCap]NoMachine(K)<=MCapacity(K);
 );

 !Labor Need;
 @FOR( LaborHour(J):
  [LaborLmt] @SUM( PRODUCT(I):NoProduct(I)*LabP(J,I))<=NoLabor(J);
 );

!Subject to labor capacity;
 @FOR( LaborHour(J):
  [LaborCap]NoLabor(J)<=LCapacity(J);
 );


 ! We want to maximize the profit;
 [OBJECTIVE] MAX = @SUM( PRODUCT(I):NoProduct(I)*PRICE(I))-
    @SUM(LaborHour(J): Lcost(J)*NoLabor(J));

END
```

OUTPUT FROM LINGO:

```
    Global optimal solution found.
   Objective value:                              15433.33
   Total solver iterations:                             5

   Model Title: Problem12
```

Variable	Value	Reduced Cost
NOPRODUCT(ONE)	16.66667	0.000000
NOPRODUCT(TWO)	50.00000	0.000000
NOPRODUCT(THR)	0.000000	1.629630
NOPRODUCT(FOUR)	33.33333	0.000000
PRICE(ONE)	300.0000	0.000000
PRICE(TWO)	260.0000	0.000000
PRICE(THR)	220.0000	0.000000
PRICE(FOUR)	180.0000	0.000000
NOLABOR(LABOR1)	583.3333	0.000000
NOLABOR(LABOR2)	650.0000	0.000000
LCAPACITY(LABOR1)	600.0000	0.000000
LCAPACITY(LABOR2)	650.0000	0.000000
LCOST(LABOR1)	8.000000	0.000000
LCOST(LABOR2)	6.000000	0.000000
NOMACHINE(M1)	700.0000	0.000000
NOMACHINE(M2)	500.0000	0.000000
MCAPACITY(M1)	700.0000	0.000000
MCAPACITY(M2)	500.0000	0.000000
LABP(LABOR1, ONE)	8.000000	0.000000
LABP(LABOR1, TWO)	5.000000	0.000000
LABP(LABOR1, THR)	4.000000	0.000000
LABP(LABOR1, FOUR)	6.000000	0.000000
LABP(LABOR2, ONE)	7.000000	0.000000
LABP(LABOR2, TWO)	8.000000	0.000000
LABP(LABOR2, THR)	7.000000	0.000000
LABP(LABOR2, FOUR)	4.000000	0.000000
MACHP(M1, ONE)	11.00000	0.000000
MACHP(M1, TWO)	7.000000	0.000000
MACHP(M1, THR)	6.000000	0.000000
MACHP(M1, FOUR)	5.000000	0.000000
MACHP(M2, ONE)	4.000000	0.000000
MACHP(M2, TWO)	6.000000	0.000000
MACHP(M2, THR)	5.000000	0.000000
MACHP(M2, FOUR)	4.000000	0.000000

Row	Slack or Surplus	Dual Price
MACHINELMT(M1)	0.000000	10.51852
MACHINELMT(M2)	0.000000	6.222222
MACHINECAP(M1)	0.000000	10.51852
MACHINECAP(M2)	0.000000	6.222222
LABORLMT(LABOR1)	0.000000	8.000000
LABORLMT(LABOR2)	0.000000	13.62963
LABORCAP(LABOR1)	16.66667	0.000000
LABORCAP(LABOR2)	0.000000	7.629630
OBJECTIVE	15433.33	1.000000

Ranges in which the basis is unchanged:

	Objective Coefficient Ranges		
Variable	Current Coefficient	Allowable Increase	Allowable Decrease
NOPRODUCT(ONE)	300.0000	28.00000	44.00000
NOPRODUCT(TWO)	260.0000	37.33333	1.725490

NOPRODUCT(THR)	220.0000	1.629630	INFINITY
NOPRODUCT(FOUR)	180.0000	8.000000	8.615385
NOLABOR(LABOR1)	-8.000000	4.478261	3.612903
NOLABOR(LABOR2)	-6.000000	INFINITY	7.629630
NOMACHINE(M1)	0.0	INFINITY	10.51852
NOMACHINE(M2)	0.0	INFINITY	6.222222

Righthand Side Ranges

Row	Current RHS	Allowable Increase	Allowable Decrease
MACHINELMT(M2)	0.0	9.677419	46.15385
MACHINECAP(M1)	700.0000	14.06250	112.5000
MACHINECAP(M2)	500.0000	9.677419	46.15385
LABORLMT(LABOR1)	0.0	583.3333	16.66667
LABORLMT(LABOR2)	0.0	47.36842	9.782609
LABORCAP(LABOR1)	600.0000	INFINITY	16.66667
LABORCAP(LABOR2)	650.0000	47.36842	9.782609
MACHINELMT(M1)	0.0	14.06250	112.5000

(a) Since

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
NOPRODUCT(THR)	220.0000	1.629630	INFINITY

When the price increases $1.62963, the optimal basis will change, then product three would be produced.

(b) The price of product 1 decreases by $10, since the Allowable Decrease is $44, so the optimal basis is still unchanged.

$OBJ_{new} = OBJ_{old} + \Delta price * NOPRODUCT(ONE) = 15433.33 + (-10) * 16.66667 = 15266.66$

(c) The extra hour for both machines: check the Allowable Increases for both machines are 14.06250, 9.677419, respectively. The corresponding dual prices are 10.51852, 6.222222, which are the company would paid for one additional hour for each machine.

(d) The extra hour for both types of labor: check the Allowable Increases for both types of labor are 16.66667, 9.782609, respectively. The corresponding dual prices are 0, 7.629630, which are the company would paid for one additional hour for each type of labor.

(e) Since the Allowable Increase for skilled labor hour is infinite, so the optimal basis is still unchanged.

$OBJ_{new} = OBJ_{old} + \Delta skilledHour * \text{Dual Price} = 15433.33 + (100) * 0 = 15433.33$. It is unchanged.