IE312 F09 Hw1 Assignment

Due on Friday, October 9th

Chapter 4.5 Problem 2 (p149)
Chapter 4.6 Problem 4 (p151)
Chapter 4.7 Problem 5 (p154)
Chapter 4.8 Problem 5 (p158)

Chapter 4.5 Problem 2 (p149)

Soln:

Max $z = 2x_1 + 3x_2$

st. $x_1 + 2x_2 \leq 6$

$2x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0$

Standard Form

Max $z = 2x_1 + 3x_2$

st. $x_1 + 2x_2 + x_3 + 0x_4 = 6$

$2x_1 + x_2 + 0x_3 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

Iteration 1

	X_1	X_2	X_3	X_4	RHS(b)
Max	2	3	0	0	0
Row 1	1	2	1	0	6
Row 2	2	1	0	1	8
bfs $\overline{X}^{(0)}$	N 0	N 0	B 6	B 8	
$\Delta\overline{X}_1$	1	0	-1	-2	$\overline{C}\Delta\overline{X}_1 = 2$
$\Delta\overline{X}_2$	0	1	-2	-1	$\overline{C}\Delta\overline{X}_2 = 3$

Increasing X_2 would be more beneficial than X_1. Let X_2 enter the basis. $\lambda^{(1)} = \min\{6/2=3, 8/1=8\} = 3$

$\overline{X}^{(1)} = \overline{X}^{(0)} + \lambda^{(1)}\Delta\overline{X}_2$

$\overline{X}^{(1)} = (0, 0, 6, 8) + 3*(0, 1, -2, -1) = (0, 3, 0, 5)$

$z^{(1)} = 9$

Iteration2

	X_1	X_2	X_3	X_4	RHS(b)
Max	2	3	0	0	0
Row 1	1	2	1	0	6
Row 2	2	1	0	1	8
bfs $\overline{X}^{(1)}$	N 0	B 3	N 0	B 5	
$\Delta\overline{X}_1$	1	-0.5	0	-1.5	$\overline{C}\,\Delta\overline{X}_1 = 0.5$
$\Delta\overline{X}_3$	0	-0.5	1	0.5	$\overline{C}\,\Delta\overline{X}_3 = 0$

Increasing X_1 would be more beneficial than X_3. Let X_1 enter the basis. $\lambda^{(2)} = min\{3/0.5=6, 5/1.5=3.333\} = 3.333$

$\overline{X}^{(2)} = \overline{X}^{(1)} + \lambda^{(2)}\,\Delta\overline{X}_1$
$\overline{X}^{(2)} = (0, 3, 0, 5) + 3.333*(1, -0.5, 0, -1.5) = (3.333, 1.3333, 0, 0)$
$z^{(2)} = 10.6667$

Iteration3

	X_1	X_2	X_3	X_4	RHS(b)
Max	2	3	0	0	0
Row 1	1	2	1	0	6
Row 2	2	1	0	1	8
bfs $\overline{X}^{(2)}$	B 3.3333	B 1.3333	N 0	N 0	
$\Delta\overline{X}_3$	1/3	-2/3	1	0	$\overline{C}\,\Delta\overline{X}_3 = -2.667$
$\Delta\overline{X}_4$	-2/3	-1/3	0	1	$\overline{C}\,\Delta\overline{X}_4 = -1.333$

The two moving directions only decrease z, so stop. Z = 10.667 is the optimal objective value when $\overline{X} = (3.333, 1.3333, 0, 0)$.

Method 2:

Iteration 1

	X_1	X_2	X_3	X4	RHS
Max	2	3	0	0	0
Row 1(X_3)	1	2	1	0	6
Row 2 (X_4)	2	1	0	1	8

Since $Z = 2X_1 + 3X_2$, increasing X_2 would be more beneficial than X_1. Let X_2 enter the basis. $\lambda = min\{6/2=3, 8/1=8\} = 3$, so X_3 will leave the basis.

Iteration 2

	X_1	X_2	X_3	X4	RHS
Max	0.5	0	-1.5	0	-9
Row 1(X_2)	0.5	1	0.5	0	3
Row 2(X_4)	1.5	0	-0.5	1	5

Since Z = 9 + 0.5X_1 -1.5 X_3, increasing X_1 would be more beneficial than X_3. Let X_1 enter the basis. λ = *min*{3/0.5=6, 5/1.5=3.333} = 3.333. So X_4 will leave the basis.

Iteration 3

	X_1	X_2	X_3	X4	RHS
Max	0	0	-1.33333	-0.33333	-10.6666667
Row 1(X_1)	0	1	0.666667	-0.33333	1.333333333
Row 2(X_2)	1	0	-0.33333	0.666667	3.333333333

Now Z = 10.667- 1.33X_3 -0.33 X_4, we cannot increase any variables to make Z increase. So stop here. Z = 10.667 is the optimal objective value when $\overline{X}$ = (3.333, 1.3333, 0, 0).

Chapter 4.6 Problem 4 (p151)

Soln:

Min $z = -3x_1 + 8x_2$

st.

$$4x_1 + 2x_2 \leq 12$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1 , x_2 \geq 0$$

Standard Form

Max $z = -3x_1 + 8x_2$

st.

$$4x_1 + 2x_2 + x_3 + 0x_4 = 12$$
$$2x_1 + 3x_2 + 0x_3 + x_4 = 6$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Iteration 1

	X_1	X_2	X_3	X_4	RHS(b)
Min	-3	8	0	0	0
Row 1	4	2	1	0	12
Row 2	2	3	0	1	6
bfs $\overline{X}^{(0)}$	N 0	N 0	B 12	B 6	
$\Delta\overline{X}_1$	1	0	-4	-2	$\overline{C}\Delta\overline{X}_1 = -3$
$\Delta\overline{X}_2$	0	1	-2	-3	$\overline{C}\Delta\overline{X}_2 = 8$

Increasing X_1 would decrease Z. Let X_1 enter the basis. $\lambda^{(1)}$ = *min*{12/4, 6/2=3} = 3

$$\overline{X}^{(1)} = \overline{X}^{(0)} + \lambda^{(1)} \Delta\overline{X}_2$$

$\overline{x}^{(1)} = (0, 0, 12, 6) + 3*(1, 0, -4, -2) = (3, 0, 0, 0)$
$z^{(1)} = -9$

Iteration2

	X_1	X_2	X_3	X_4	RHS(b)
Max	-3	8	0	0	0
Row 1	4	2	1	0	12
Row 2	2	3	0	1	6
bfs $\overline{X}^{(1)}$	B 3	N 0	N 0	B 0	
$\Delta\overline{X}_2$	-0.5	1	0	-2	$\overline{C}\Delta\overline{X}_2 = 9.5$
$\Delta\overline{X}_3$	-0.25	0	1	0.5	$\overline{C}\Delta\overline{X}_4 = 0.75$

The two moving directions only increase z, so stop. Z = -9 is the optimal objective value when $\overline{x}$ = (3, 0, 0, 0).

Method 2:
Iteration 1

	X_1	X_2	X_3	X4	RHS
Max	-3	8	0	0	0
Row 1(X_3)	4	2	1	0	12
Row 2 (X_4)	2	3	0	1	6

Since $Z = -3X_1 + 8X_2$, increasing X_1 would decrease Z. Let X_1 enter the basis. λ = *min*{12/4, 6/2=3} = 3, so X_3 will leave the basis.

Iteration 2

	X_1	X_2	X_3	X4	RHS
Max	0	9.5	0.75	0	9
Row 1(X_1)	1	0.5	0.25	0	3
Row 2 (X_4)	0	2	-0.5	1	0

Now $Z = -9 + 9.5X_2 + 0.75X_3$, we cannot increase any variables to make Z decrease. So stop here. Z = -9 is the optimal objective value when $\overline{x}$ = (3, 0, 0, 0).

Chapter 4.7 Problem 5 (p154)

Soln:

$$\begin{aligned} \text{Max} \quad & z = 2x_1 + 2x_2 \\ \text{st.} \quad & x_1 + x_2 \le 6 \\ & 2x_1 + x_2 \le 13 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Standard Form

$$\text{Max} \quad z = 2x_1 + 2x_2$$

st. $x_1 + x_2 + x_3 + 0x_4 = 6$
$2x_1 + x_2 + 0x_3 + x_4 = 13$
$x_1, x_2, x_3, x_4 \geq 0$

Iteration 1

	X_1	X_2	X_3	X_4	RHS(b)
Max	2	2	0	0	0
Row 1	1	1	1	0	6
Row 2	2	1	0	1	13
bfs $\overline{X}^{(0)}$	N 0	N 0	B 6	B 12	
$\Delta\overline{X}_1$	1	0	-1	-2	$\overline{C}\Delta\overline{X}_1 = 2$
$\Delta\overline{X}_2$	0	1	-1	-1	$\overline{C}\Delta\overline{X}_2 = 2$

Increasing X_1 and X_2 all increase the objective value, and they have the same effect, so we select X_1 arbitrary. Let X_1 enter the basis. $\lambda^{(1)} = min\{6/1=6, 13/2=7.5\} = 6$.

$\overline{X}^{(1)} = \overline{X}^{(0)} + \lambda^{(1)} \Delta\overline{X}_2$
$\overline{X}^{(1)} = (0, 0, 6, 13) + 6*(1, 0, -1, -2) = (6, 0, 0, 1)$
$z^{(1)} = 12$

Iteration 2

	X_1	X_2	X_3	X_4	RHS(b)
Max	2	2	0	0	0
Row 1	1	1	1	0	6
Row 2	2	1	0	1	13
bfs $\overline{X}^{(1)}$	B 6	N 0	N 0	B 1	
$\Delta\overline{X}_2$	-1	1	0	1	$\overline{C}\Delta\overline{X}_2 = 0$
$\Delta\overline{X}_3$	-1	0	1	2	$\overline{C}\Delta\overline{X}_3 = -2$

The two moving directions cannot increase z, so stop. Z = 12 is the optimal objective value when $\overline{X} = (6, 0, 0, 1)$.
We notice that the direction $\Delta\overline{X}_2$ cannot decrease z either. If moving along this direction, the optimal objective value keeps the same, but the optimal points change. So we have infinite optimal solutions. Now find the other optimal basic point. Let X_2 enter the basis. $\lambda^{(2)} = min\{6/1=6\} = 6$.

$\overline{X}^{(2)} = \overline{X}^{(1)} + \lambda^{(2)} \Delta\overline{X}_1$
$\overline{X}^{(2)} = (6, 0, 0, 1) + 6*(-1, 1, 0, 1) = (0, 6, 0, 7)$
$z^{(2)} = 12$

So the number of optimal solution is infinite, Optimal solutions are $c*(6, 0, 0, 1) + (1-c)*(0, 6, 0, 7)$, where c is a scalar, $0 \leq c \leq 1$.

Method 2:

Iteration 1

	X_1	X_2	X_3	X4	RHS
Max	2	2	0	0	0
Row 1(X_3)	1	1	1	0	6
Row 2 (X_4)	2	1	0	1	13

Since $Z = 2X_1 + 2X_2$, increasing both X_1 and X_2 would be beneficial. They are tied, we choose one arbitrary. Let X_1 enter the basis. $\lambda = min\{6/1, 13/2\} = 6$, so X_3 will leave the basis.

Iteration 2

	X_1	X_2	X_3	X4	RHS
Max	0	0	-2	0	-12
Row 1 (X_1)	1	1	1	0	6
Row 2 (X_4)	0	-1	-2	1	1

Now $Z = 12 - 2X_3$, we cannot increase any variables to make Z increase. So stop here. Z = 12 is the optimal objective value when $\bar{x} = (6, 0, 0, 1)$.

However, notice that if we increase X_2, the objective value would keep the same. Let X into the basis, then X leave the basis. After a pivot, we get the table as follows.

	X_1	X_2	X_3	X4	RHS
Max	0	0	-2	0	-12
Row 1 (X_2)	1	1	1	0	6
Row 2 (X_4)	1	0	-1	1	7

Another optimal solution we get (0,6,0,7). The linear segment between these two points is also optimal points. So the number of optimal solution is infinite. The optimal solutions are c*(6, 0, 0, 1) + (1-c)*(0,6,0,7), where c is a scalar, $0 \le c \le 1$.

Chapter 4.8 Problem 5 (p158)

Soln:

Max $z = x_1 + 2x_2$

st.
$$-x_1 + x_2 \le 2$$
$$-2x_1 + x_2 \le 1$$
$$x_1 , x_2 \ge 0$$

Standard Form

Max $z = x_1 + 2x_2$

st.
$$-x_1 + x_2 + x_3 + 0x_4 = 2$$
$$-2x_1 + x_2 + 0x_3 + x_4 = 1$$
$$x_1, x_2, x_3, x_4 \ge 0$$

Iteration 1

	X_1	X_2	X_3	X_4	RHS(b)
Max	1	2	0	0	0
Row 1	-1	1	1	0	2
Row 2	-2	1	0	1	1
bfs $\overline{X}^{(0)}$	N 0	N 0	B 2	B 1	
$\Delta\overline{X}_1$	1	0	1	2	$\overline{C}\Delta\overline{X}_1 = 1$
$\Delta\overline{X}_2$	0	1	-1	-1	$\overline{C}\Delta\overline{X}_2 = 2$

Notice that the direction of $\Delta\overline{X}_1$ increase the objective value, but all the numbers are positive, which means no constraint could break it. So it will go infinite, which mean the optimal solution is unbounded.

Method 2:

Iteration 1

	X_1	X_2	X_3	X4	RHS
Max	1	2	0	0	0
Row 1	-1	1	1	0	2
Row 2	-2	1	0	1	1

Notice that increasing X_1 will increase the objective value Z, since the coefficients of X_1 are negative in all the constraints, when X_1 increases, the constraints may need other variables to increase rather than decrease, so other variables would not reach zero, which breaks X_1 to become bigger and bigger.

OR

Iteration 1

	X_1	X_2	X_3	X4	RHS
Max	1	2	0	0	0
Row 1(X_3)	-1	1	1	0	2
Row 2(X_4)	-2	1	0	1	1

Since $Z = X_1 + 2X_2$, increasing X_2 would be more beneficial than X_1. Let X_2 enter the basis. $\lambda = min\{2/1=2, 1/1=2\} = 1$, so X_4 will leave the basis.

Iteration 2

	X_1	X_2	X_3	X4	RHS
Max	5	0	0	-2	-2
Row 1(X_3)	1	0	1	-1	1
Row 2(X_2)	-2	1	0	1	1

Now $Z = 2 + 5X_1 - 2X_4$, we can increase X_1 to make Z increase. $\lambda = min\{1/1\} = 1$, so X_3 will leave the basis.

Iteration 3

	X_1	X_2	X_3	X4	RHS
Max	0	0	-5	3	-7
Row 1(X_1)	1	0	1	-1	1
Row 2(X_2)	0	1	2	-1	3

Now $Z = 7 - 5X_3 + 3X_4$, we can increase X_4 to make Z increase. Since the coefficient of X_4 are all negative,

The constraints:

st. $x_1 + x_3 - x_4 = 1$

$x_2 + 2x_3 - x_4 = 3$

$x_1, x_2, x_3, x_4 \geq 0$

When X_4 increasing, other variables also increase. Then the objective value becomes bigger and bigger. No any constraints could break it. So stop. The optimal solution is unbounded.