Spring 2008

Statistics 580
Project #1(100 points)

This project is in two parts. Use R code to answer the first part and C code for the second. Need to provide code used via e-mail in addition to the printed versions to be attached to the solutions turned-in. You should mail the code to me packaged as two `.tar.gz` files. Read all instructions below before you begin.

Part1

Compute the following integrals using the specified Monte Carlo methods in each case. Wherever possible, compute the variance of the crude MC estimator (either via MC or theoretically) and calculate the variance reduction.

a. Evaluate $\int_0^\infty sin^2(x)e^{-x/2}dx$ using antithetic exponential random variates.

b. Estimate the variance of $logistic(0, 1)$ distribution using antithetic $logistic(0, 1)$ random variates.

c. Estimate $P(Z > 4.5)$ to 4-digit accuracy using importance sampling with nonstandardized weights, where $Z \sim N(0, 1)$. Use the exponential distribution with mean 1, left truncated at 4.5, as the importance density.

d. Estimate $E(X^2)$ when X has density that is $\propto e^{-|x|^3/3}$ with support $(-\infty, \infty)$ using importance sampling with standardized weights.

e. Repeat part d.) using importance sampling with standardized weights and a control variate.

f. Evaluate $\int_0^\infty g(x)f(x)dx$ where $g(x) = x/(1+x)$ and $f(x) = \sqrt{\frac{2}{\pi}}\ \exp^{-x^2/2}\ I_{[0,\infty)}(x)$, the folded standard normal density, using crude MC. Use an acceptance-rejection method to generate from this distribution.

g. Repeat part f.) using any one or more variance reduction techniques applicable.

Print a table listing the method, the estimate, it's standard error, and the variance reduction over the crude MC estimator (for each case when possible).

Part 2

Consider testing the hypotheses $H_0 : \lambda = 3$ versus $H_a : \lambda > 3$ using a random sample of size n from a Poisson distribution with mean λ. Application of the central limit theorem would suggest rejecting H_0 at level α when $Z \geq z_{1-\alpha}$, where $Z = \frac{\bar{X}-3}{\sqrt{3/n}}$ and $z_{1-\alpha}$ is the $100(1-\alpha)^{\text{th}}$ percentile of the standard normal distribution.

(a) Estimate the size of this test (i.e., the type I error rate) using three Monte Carlo approaches: crude and importance sampling with unstandardized and standardized weights. Provide a standard error for each estimate.

`Note:` For the importance sampling approaches, select a Poisson density under H_a : as the importance function. Examine several choices of λ under H_a : for improving variance reduction.

(b) Repeat the above estimation procedures for $\alpha = .1, .05, 01$ and $n = 12, 16, 25, 30$ and report results in an appropriate table.

`Note:` See Example 3 (p. 12 of the MC note) for some hints on how to set-up this problem.

Instructions:

1. You are on your honor to do your own work on this project. Do not discuss anything with others in class.
2. You may ask questions or clarifications (in class, in my office or by e-mail) from me, and if it is relevant, I will discuss them in class.
3. Grading of the project will include credit for the overall plan for the solution to each problem, the the organization of code, and the presentation of esults.
4. For the C part, you may use `Rmath` library or `GSL` but not both. It is recommended that you use a `makefile` because it will simplify the housekeeping when several functions are compiled many times.
5. You must provide the seeds to the random number generators used so that I may replicate all results reported.
6. You are required to use the `Linux` version of R and save the .RData, source code and results in folders in your afs share so that they are accessible if any revisons are required.
7. Use Monte Carlo samples of size $N = 1000$ for all simulations.

Due Thursday, March 13, 2008 (tentative)