STATISTICS 401C **Fall 2008**

Laboratory Assignment 3

1. A company has determined that an average of 7.5% of a certain manufactured item will be damaged during shipment. Consider a shipment consisting of 140 of these items.

   (a) Verify that the conditions for using the Normal approximation to the Binomial are satisfied.

   (b) Use the Normal approximation to the Binomial (with the continuity correction) to find the approximate probability that during this shipment

      i. at least 20 items will be damaged.

      ii. no more than 5 items will be damaged.

      iii. exactly 10 items will be damaged.

   (c) Calculate the exact answer to part (b)iii using the Binomial probability function.

2. A particular brand of cement is sold in 50 lb bags. Suppose that the actual net weight X of a randomly chosen bag of cement of this brand is Normally distributed with mean $\mu = 50$ lbs. and standard deviation $\sigma = 1$ lb.

   (a) A bag is considered underfilled if it weighs less than 49.0 lbs. What is the probability that a randomly selected bag is underfilled ? Show your work.

   (b) A bag will meet the standard established by a consumer organization if it weighs between 49.75 and 50.75 lbs. What is the probability that a randomly selected bag will meet this standard? Show your work.

   (c) In a random sample of cement bags only 1% of the bags are heavier than a certain weight. What, approximately, is this weight? Show your work.

   (d) How many bags would you expect to be underfilled, approximately, in a randomly selected lot of 5000 cement bags of this brand? You must derive your answer using theory you learned in Chapter 4 regarding the Binomial random variable.

3. Suppose that the tar content of cigarettes of a certain brand is distributed with mean $\mu = 10$ mg. and standard deviation $\sigma = 2.4$ mg. A random sample of $n = 36$ cigarettes from this brand are tested. Let $\overline{Y}$ be the random variable that corresponds to the mean of this sample.

(a) Describe the approximate distribution of $\overline{Y}$ according to the Central Limit Theorem.

(b) Find $P(\overline{Y} > 11)$ assuming the distribution from part (a) for $\overline{Y}$. Show your work.

(c) Find $P(9.4 < \overline{Y} < 10.8)$ assuming the distribution from (a) for $\overline{Y}$. Show your work.

(d) Find k such that $P(\overline{Y} > k) = 0.05$ assuming the distribution from (a) for $\overline{Y}$. Show your work.

(e) Find c such that $\overline{Y}$ is in the interval $(10 - c, 10 + c)$ with a probability of 0.95. Show your work.

4. A quality control inspector is investigating the width, Y, of door latches used on new cars. Suppose that the distribution of Y is assumed to be Normal with mean 500 millimeters and standard deviation 2 millimeters (assuming that the width of latches is measured on a continuous scale).

   (a) If a latch is acceptable only if its width is within 0.5% of the population mean, what is the probability that a randomly chosen specimen is acceptable? Show your work.

   (b) What is the sampling distribution $\overline{Y}$ of random samples of size 100 from a population of these latches? State the mean and standard deviation of $\overline{Y}$ clearly.

   (c) Compute the probability that the mean width of a random sample of 100 latches exceeds 500.1 millimeters.

   (d) If the acceptable range is as in part (a) and the width of each of 10 randomly selected specimens is independently determined, what is the probability that at least 9 out 10 are acceptable. Show your work. [*Hint:* First define the acceptable number in a sample of size 10 as a Binomial random variable.

`Due Thurssday, September 25, 2008 (turn-in during lab)`