For problems 1 and 2, consider again the population of % people with Graduate Degrees in a population of cities given in Lab Assignment #2.

1. From three samples of sizes 12, 24, and 36, respectively were drawn by your instructor from the population and the following sample means and variances were calculated:

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size n</td>
<td>12</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>Sample mean ( \bar{y} )</td>
<td>8.19</td>
<td>9.80</td>
<td>10.01</td>
</tr>
<tr>
<td>Sample Std. Dev. s</td>
<td>4.60</td>
<td>4.33</td>
<td>4.81</td>
</tr>
</tbody>
</table>

(a) For these samples compute confidence intervals (by hand) for the population mean \( \mu \) as specified below. Use the Student’s \( t \) distribution when computing the intervals. Show all your work.

**Sample 1**  
90% confidence interval

\[
8.19 \pm t_{0.05, 11} \times \frac{4.60}{\sqrt{12}}, \text{ where } t_{0.05, 11} = 1.796
\]

\[
(5.81, 10.57)
\]

**Sample 2**  
95% confidence interval

\[
9.80 \pm t_{0.025, 23} \times \frac{4.33}{\sqrt{24}}, \text{ where } t_{0.025, 23} = 2.069
\]

\[
(7.97, 11.63)
\]

**Sample 3**  
99% confidence interval

\[
10.01 \pm t_{0.005, 35} \times \frac{4.81}{\sqrt{36}}, \text{ where } t_{0.005, 35} = 2.724
\]

\[
(7.82, 12.19)
\]
(b) For the samples compute the t-test statistic and conduct tests of hypotheses as specified below. State your decisions clearly. Also, compute the level of significance (p-value) for each test to the nearest value possible using the t-table (Table 2 in your textbook). Show all your work.

Sample 1  Test \( H_0 : \mu = 9 \) vs. \( H_a : \mu \neq 9 \) using \( \alpha = 0.1 \)

\[
\begin{align*}
t_c &= \frac{8.19 - 9}{4.60 / \sqrt{12}} = -0.61
\end{align*}
\]

\( t_{.05, 11} = 1.796 \Rightarrow R. R. \ t \leq 1.796 \)

\[p\text{-value} = P(T_{11} < -0.61) + P(T_{11} > 0.61) \approx 2 \times (0.25, 0.4) \]

Sample 2  Test \( H_0 : \mu \leq 8 \) vs. \( H_a : \mu > 8 \) using \( \alpha = 0.05 \)

\[
\begin{align*}
t_c &= \frac{9.80 - 8}{4.33 / \sqrt{24}} = 2.037
\end{align*}
\]

\( t_{.05, 23} = 1.714 \Rightarrow R. R. \ t > 1.714 \)

\[p\text{-value} = P(T_{23} > 2.037) \Rightarrow (0.025, 0.05) \]

Sample 3  Test \( H_0 : \mu \geq 11 \) vs. \( H_a : \mu < 11 \) using \( \alpha = 0.01 \)

\[
\begin{align*}
t_c &= \frac{10.01 - 11}{4.81 / \sqrt{32}} = -1.235
\end{align*}
\]

\( t_{.01, 35} = 2.438 \Rightarrow R. R. \ t \leq -2.438 \)

\[p\text{-value} = P(T_{35} < -1.235) = P(T_{35} > 2.435) \Rightarrow (0.1, 0.25) \]

2. Links to the JMP data tables of the three samples used in Problem 1 are available on the lab assignment page.

(a) Use JMP to construct 90%, 95%, and 99% confidence intervals for the population mean \( \mu \) using each of the three samples, respectively.

(b) Use JMP to compute the t-test statistic and p-value to perform the respective tests of the population mean \( \mu \) as per Problem 1(b), using each of the three samples, respectively.

Assume \( \sigma^2 \) is unknown. Save the output from the JMP analysis each sample as a single page in Word. Do not need anything in the output other than the relevant plots, Moments and the CI's, the t-statistics, and the p-values. Circle the p-values for each test and use them to state you decisions from the tests using \( \alpha \) values given for each case in Problem 1(b) and write these by hand in each page of output.
3. To test the durability of a new paint for white center lines, a highway department painted test strips across heavily travelled roads in 8 different locations, and electronic counters showed that they deteriorated after having driven over by 14.26, 16.78, 13.65, 10.83, 12.64, 13.37, 16.20, and 14.94 million vehicles, respectively. Construct a 95% confidence interval for the mean volume of traffic (in millions of vehicle crossings) this paint can withstand before it deteriorates. Note: Use hand calculation for parts (a) to (e) and show work. Use a JMP analysis of the data for answering parts (f) and (g). Turn in the JMP output as a single page in Word.

(a) Prepare a table of quantiles of vehicle crossings data and the standard normal distribution for \( p \) values \( (i - .5)/8, \ i = 1, 2, ..., 8 \).

<table>
<thead>
<tr>
<th>i</th>
<th>((i-.5)/8)</th>
<th>Ordered Data</th>
<th>z-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0625</td>
<td>10.83</td>
<td>-1.535</td>
</tr>
<tr>
<td>2</td>
<td>0.1875</td>
<td>12.64</td>
<td>-0.885</td>
</tr>
<tr>
<td>3</td>
<td>0.3125</td>
<td>13.37</td>
<td>-0.490</td>
</tr>
<tr>
<td>4</td>
<td>0.4375</td>
<td>13.65</td>
<td>-0.16</td>
</tr>
<tr>
<td>5</td>
<td>0.5625</td>
<td>14.26</td>
<td>0.16</td>
</tr>
<tr>
<td>6</td>
<td>0.6875</td>
<td>14.94</td>
<td>0.49</td>
</tr>
<tr>
<td>7</td>
<td>0.8125</td>
<td>16.20</td>
<td>0.885</td>
</tr>
<tr>
<td>8</td>
<td>0.9375</td>
<td>16.78</td>
<td>1.535</td>
</tr>
</tbody>
</table>

(b) Draw the Normal Probability plot using the quantile pairs in part (a). Does it appear that the target population of volume of traffic before the paint deteriorates is approximately Normal? Explain.

Yes, the values fall in a straight line indicating that the data drawn from a normal distribution.
(c) Assume that the population of the volume of vehicle crossings is adequately modeled by a Normal distribution. Construct and interpret a 95% confidence interval for mean volume of traffic the paint can withstand before it deteriorates.

\[
\bar{Y} = \frac{112.67}{8} = 14.08 \quad \Sigma Y_i^2 = 1612.70 \quad S^2 = \frac{\Sigma Y_i^2 - (\Sigma Y_i)^2}{n-1} = \frac{1612.70 - (112.67)^2}{8-1} = 3.677
\]

95% CI: \( \bar{Y} \pm t_{0.05/2} \cdot \frac{S}{\sqrt{n}} \)

\[= 14.08 \pm 2.365 \cdot 0.67986 \]

95% of the time, a paint can withstand between 12.472 and 15.688 million cars before it needs repainting.

(d) If from past studies of this type it is known that the population standard deviation \( \sigma \) may be assumed to be approximately 2 million crossings, what is the minimum sample size needed to estimate the population mean volume of traffic within ±1 million with 95% confidence? (Assume that the population is normal for this part.)

\[n = \left( \frac{Z_{0.96}}{E} \right)^2 = \left( \frac{1.96}{1} \right)^2 = 15.366 \Rightarrow 16\]

(e) Does the data contradict the paint manufactures claim that this paint can withstand 15 million vehicle crossings before it deteriorates? That is, can you conclude that the mean volume of traffic before paint deteriorates is actually less than 15 million vehicles? (Use \( \alpha = .05 \) to conduct an appropriate statistical test and show work).

\[H_0: \mu \geq 15 \quad H_a: \mu < 15 \quad \alpha = 0.05\]

\[t = \frac{\bar{Y} - \mu}{S/\sqrt{n}} = \frac{14.08 - 15}{1.925/\sqrt{8}} = \frac{-0.92}{0.67986} = -1.35\]

R. R.: \( t \leq -t_{\alpha}, n-1 = -t_{0.05} \cdot 7 = -1.895 \)

Fail to reject \( H_0 \Rightarrow \) No sufficient evidence to indicate \( \mu < 15 \) at \( \alpha = 0.05 \) level.
(f) Extract the 95% confidence interval for the population mean volume of traffic from the JMP output. Copy it here and circle it in the JMP output.

\[ 95\% \text{ CI: } (12.47, 15.698) \]

(g) Extract the t-statistic from the JMP output for testing the hypothesis you stated in part (e). Circle it in the JMP output. State the p-value associated with this test extracted from the JMP output.

\[ t_{\text{observed}} = -1.3478 \]

\[ p\text{-value} = P(t < t_{\text{observed}}) = 0.1099. \]

Due Thursday, September 17th, 2012