Assessing Certainty of Activation or Inactivation in Test-Retest fMRI Studies

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Abstract

Functional Magnetic Resonance Imaging (fMRI) is widely used to study activation in the human brain. In most cases, data are commonly used to construct activation maps corresponding to a given paradigm. Results can be very variable, hence quantifying certainty of identified activation and inactivation over studies is important. This paper provides a model-based approach to certainty estimation from data acquired over several replicates of the same experimental paradigm. Specifically, the p-values derived from the statistical analysis of the data are explicitly modeled as a mixture of their underlying distributions; thus, unlike methodology currently in use, there is no subjective thresholding required in the estimation process. The parameters governing the mixture model are easily obtained by the principle of maximum likelihood. Further, the estimates can also be used to optimally identify voxel-specific activation regions along with their corresponding certainty measures. The methodology is applied to a study involving a motor paradigm performed on a single subject several times over a period of two months. Simulation experiments used to calibrate performance of the method are promising. The methodology is also seen to be robust in determining areas of activation and their corresponding certainties.

Key words: fMRI, quantification, intra-class correlation coefficient, maximum likelihood estimation, mixture distribution, motor task, percent overlap, true activation certainty, true inactivation certainty

1. Introduction

Functional Magnetic Resonance Imaging (fMRI) has become an extremely popular noninvasive imaging modality for understanding human cognitive and motor functions. The main goal of fMRI is to identify regions of the brain that are activated by a given stimulus or while performing some task, but high variability among replicated studies often leads to inconsistent results, causing concern among researchers (see, for instance, in Buchsbaum et al., 2005, Derrfuss et al., 2005, Ridderinkhof et al. 2004, or Uttal 2001). There are a number of factors that affect the identification of activated voxels. A typical fMRI paradigm consists of the application of a stimulus or performance of a cognitive or motor task over time. Any neural stimulus passes through the so-called hemodynamic filter (Maitra et al., 2002), resulting in a several-seconds delay before the blood-oxygen-level-dependent (BOLD) response occurs. Other factors also affect the acquired data (Genovese et al., 1997). For example, the cardiac and respiratory motion of a subject may result in physiological variation, giving rise to flow-artifacts which may need to be monitored or digitally filtered (Biswal et al., 1996). Subjects also often exhibit voluntary, involuntary and/or stimulus-correlated motion during scans (Hajnal et al., 1994). Another factor is scanner variability which is essentially controlled through effective quality control programs. Most signal differences between activated and control or resting states are small, typically on the order of 1–5% (Chen and Small, 2007), and sub-pixel motions can induce large apparent signal changes and result in the detection of false positives.

Therefore, fMRI data are subjected to image registration algorithms which align the sequence of images to sub-pixel accuracy (Wood et al., 1998). The pre-processing of data improves the quality of acquired fMRI data, but identified regions of activation still vary from one replication to the other. This variability needs to be quantified in order to determine regions of activation with precision and accuracy (McGonigle et al., 2000; Noll et al., 1997; Wei et al., 2004).

Repeatability of results across multiple studies is one way of assessing variability and measures that calibrate repeatability are called reliability measures. Many authors working in the area of fMRI image variability used the term reliability to describe the extent to which activation was consistently identified in multiple fMRI images. However, there is another, perhaps more useful, quantity of interest to practitioners: quantification of the true status of voxels identified as activated or inactivated. Measures that attempt to quantify the probability of the true status of a voxel given its identified state are more correctly termed measures of confidence or certainty even though these were also introduced, perhaps confusingly, as reliability measures by earlier authors that included me. In this paper, I will move towards adopting the nomenclature of certainty in these contexts in order to better distinguish it from simple reliability. But before proceeding further, I specify that I use the term “replication” to denote the repetition of the task or experimental condition to study variability. These replications are necessarily independent and, in the context of single-subject studies, occur on different scanning sessions, reasonably separated in time.
The issue of quantifying variability (whether reliability or certainty) of activation has interested researchers in two different frameworks. The first case involves the analysis of grouped fMRI data, which arise when multiple subjects are studied under multiple stimulus or task-performance levels, e.g., fMRI data acquired while subjecting multiple volunteers to noxious painful stimuli at several graded levels. I will refer to these stimulus or task levels as experimental conditions. The second scenario, which is the focus of this paper, is the test-retest case, where replicated fMRI data are acquired on the same subject under the same experimental condition.

For grouped fMRI data, the goal is to determine where the effect of the stimulus is larger than subject-to-subject variation. Reliability of activation in response to stimulus has been quantified in terms of the intra-class correlation (ICC), which is calculated using voxels identified as activated in each subject after thresholding separately for each combination of experimental condition and subject (Aron et al. (2006); Fernández et al. (2003); Friedman et al., (2008); Manoach et al. (2001); Meizin et al. (2000); Raemekers et al. (2007); Sprecht et al. (2003)). The ICC provides a measure of correlation or conformity between regions identified as activated in multiple subjects under two or more experimental replicates and/or conditions. Thus it is inapplicable to the test-retest framework on a single subject considered in this paper.

For test-retest, Rombouts et al. (1998) and Machielsen et al. (2000) have proposed a global reliability measure of the percent overlap in voxels identified as activated between any two experimental replications. For any two replications (say, \( j \) and \( m \)), this measure is calculated as \( R_{jm} = 2V_{jm}/(V_j + V_m) \), where \( V_{jm} \) is the number of three-dimensional image voxels identified as activated in both the \( j \)th and \( m \)th replications, and \( V_j \) and \( V_m \) represent the number of voxels identified as activated, separately in the \( j \)th and \( m \)th replicated experiments, respectively. \( R_{jm} \) takes a value between 0 and 1, representing no to perfect overlap in identified activation at the two ends of the scale.

The percent overlap measure \( R_{jm} \) provides a measurement of the agreement in activation between any two replications, but it is sensitive to the method of identifying activation, unusable for voxel-level analysis, and awkward for more than two replicates. To illustrate \( R_{jm} \) sensitivity to method, consider a procedure that liberally identifies activation (e.g., a naive testing approach with no correction for multiple testing), the denominator \( V_j + V_m \) would be large so that small disagreement in the voxels identified as activated would have very little impact on \( R_{jm} \). In contrast, small differences in \( V_{jm} \) would severely affect \( R_{jm} \) when \( V_j + V_m \) is small, as expected under a conservative method (e.g., testing with the Bonferroni correction for multiple testing). Another shortcoming is that \( R_{jm} \) is a global measure of agreement between replicated experiments giving no sense of voxel-level reliability of activation. One could compute separate \( R_{jm} \) for specific brain regions, but it will never be a high-resolution measure of activation reliability. A third concern is that \( R_{jm} \) is a reliability measure based only on the pair \((j, m)\) of experimental replicates. When there are \( M \) replicates or \( M \) studies combined in a composite meta-analysis, there are \( \binom{M}{2} \) overlap measures \( R_{jm} \) and there is no obvious way to combine them in a single measure of activation reliability. Thus, there is a need for a measure to quantify reliability or certainty of true activation at the voxel level across an arbitrary number of replicates. Ideally, such an assessment would be independent of the experimental condition and method used to identify activation.

Some more formal statistical approaches to assessing reliability in the test-retest fMRI framework have been proposed as well. Genovese et al. (1997) and Noll et al. (1997) specified probabilities that voxels were correctly or incorrectly identified as activated at particular thresholds of the test statistic to determine significance of activation for a given experimental paradigm. Their approach modeled the total frequency (out of \( M \) replications) of a voxel identified as activated at given thresholds in terms of a mixture of binomial distributions. To combine data, they assumed independence over the thresholds. All parameters, such as the mixing proportion of truly active voxels (denoted as \( \lambda \) in their work) or probability of voxels being correctly \((\pi_1)\) or incorrectly \((\pi_0)\) identified as active were assumed to be spatially independent and estimated using maximum likelihood (ML) methods. Maitra et al. (2002) extended their proposals by incorporating a more accurate model of mixtures of conditional binomial distributions, and by also generalizing \( \lambda \) to be voxel-specific. Specifically, they let \( \lambda_l \) be the probability that the \( l \)th voxel is truly active. Letting \( L \) be the number of activation threshold levels (assumed without loss of generality to be in increasing order), define \( X_l = (x_{l,1}, x_{l,2}, \ldots, x_{l,L}) \), where \( x_{l,j} \) is the number of replications for which the \( j \)th voxel is identified as activated at the \( l \)th threshold. Let \( \eta_{Al,l-1} \) (and \( \eta_{Il,l-1} \)) be the (global) probability of a truly active (correspondingly, truly inactive) voxel being identified as activated at the \( l \)th threshold, given that it has been so identified at the \((l-1)\)th threshold level. Also, let \( \pi_{Al} \) (or \( \pi_{Il} \)) be the probability that a truly active (correspondingly inactive) voxel is identified as activated at the \( l \)th threshold. Then the likelihood function for the \( l \)th voxel is provided by

\[
\lambda_l \prod_{j=1}^{L} \left( \frac{x_{l,j-1}}{x_{l,j}} \right) \eta_{Al,l-1}^{x_{l,j-1}} (1 - \eta_{Al,l-1})^{x_{l,j-1}-x_{l,j}} \\
+ (1 - \lambda_l) \prod_{j=1}^{L} \left( \frac{x_{l,j-1}}{x_{l,j}} \right) \eta_{Il,l-1}^{x_{l,j-1}} (1 - \eta_{Il,l-1})^{x_{l,j-1}-x_{l,j}}
\]

where \( x_{l,0} = L, \eta_{Al,0} = \pi_{Al} \) and \( \eta_{Al,1} = \pi_{Il,0} \). A further generalization incorporated spatial context by regularizing \( \lambda \) through a Markov Random Field component in the penalty term of the estimation process. Estimates were obtained by maximizing the penalized likelihood. Maitra et al. (2002) introduced a novel approach to quantifying certainty about the true status of voxels identified as activated/inactivated by defining a measure of reliability – the probability of a voxel identified as activated being truly active – and anti-reliability – the probability of a voxel incorrectly identified as inactivated being active. In naming these certainty measures (as mentioned earlier) they aligned them with the layman’s notion of reliability: trustworthiness of identified activation. Maitra et al. (2002) also extended Genovese et
al. (1997)’s approach to provide a voxel-specific method for choosing the optimal threshold for detecting activation by maximizing the “reliability efficient frontier” i.e., the probability of making a correct decision on the state of a voxel (whether activated or inactivated) at a given threshold. Their emphasis was on assessing certainty of activation and inactivation in a test-retest setting, but the method was also subsequently extended to grouped functional MR imaging data by Gullapalli et al. (2005).

The methodology of Genovese et al. (1997) and Maitra et al. (2002) is implemented by obtaining a test statistic and thresholding it (or more commonly, its p-value) at different levels. This is integral to obtaining the $x_i$‘s used in (1). However, there is no clear guideline to choosing the thresholds which is left to the researcher. The choice of the number $L$ and value of these thresholds is subjective and can greatly impact the reliability and certainty estimates. Too few threshold levels can result in severely biased estimates, while too many may be computationally burdensome besides having high variability in the estimates. An additional issue is the subjective choice of spacing between the thresholds, to which there is also no satisfactory answer. In this paper, we reformulate the problem in order to eliminate this requirement of threshold choice altogether. Specifically, we model the distribution of the voxel-wise p-value of the test statistic in terms of a mixture of two distributions. The first component of the mixture is the standard uniform density corresponding to the distribution of the p-value under the null hypothesis of no activation. The second is the distribution of the p-value when there is activation. While a mixture of beta distributions is sometimes used to approximate this latter distribution (Pounds and Morris, 2003; Allison et al., 2002), we note that it is possible to derive exact distributions in many standard scenarios, such as t-tests. Also, the mixing proportion of the mixture component representing the distribution of the p-value under activation is the same as the $\lambda$ in Genovese et al. (1997) or Maitra et al. (2002). Estimation is done using ML. Once again, optimal cut-offs can be estimated by maximizing the reliability efficient frontier. To better reflect the fact that we are quantifying certainty in the true status of voxels identified as activated and inactivated, we rename the erstwhile reliability measure as the true activation certainty and the awkwardly-termed anti-reliability measure in terms of unity, calling the latter the true inactivation certainty. Estimates for both measures are also provided. The methodology is applied to an experiment involving a motor paradigm that was replicated on the same subject twelve times over the course of two months. The performance of the suggested method is also validated via simulation experiments over a range of replication sizes. Further, we randomly subdivide the dataset into two subsets of six replications, and study the robustness of the identified activation and the corresponding true activation and inactivation certainties. We conclude with some discussion.

2. Theory

The p-value of a test statistic $T_o$ is the probability, under the null distribution, of obtaining a more extreme value (in the direction of the alternative) than $T_o$. For a one-sided t-test for the null hypothesis $H_0 : \beta = 0$ against the alternative $H_o : \beta > 0$ with $\beta$ as the regression coefficient of a general linear model fit to the time series at a voxel, this is given by $\Pr(t_o \geq T_o)$, where $\Pr$ abbreviates probability and $t_v$ denotes a $t$-distributed random variable with $v$ degrees of freedom and cumulative distribution and density functions $\Psi_v(\cdot)$ and $\psi_v(\cdot)$ respectively.

Let $P_i$ be the p-value at the $i$th voxel of the t-statistic with $v$ degrees of freedom. Under the (null) hypothesis of no true activation at a voxel, the p-value follows the standard uniform distribution. To see this,

$$
\Pr(P_i \leq p \mid H_0) = \Pr[\Pr(t_v \geq T_o) \leq p] = \Pr[\Psi_v(T_o) \geq 1 - p] = 1 - \Pr[T_o \leq \Psi_v^{-1}(1 - p)] = 1 - \Psi_v^{-1}(1 - p) = p.
$$

On the other hand, under the alternative one-sided hypothesis that the voxel is truly activated, we get

$$
\Pr(P_i \leq p \mid H_o) = \Pr[\Pr(t_v \geq T_o) \leq p \mid t_v, \alpha] = \Pr[\Psi_v(T_o) \geq 1 - p \mid t_v, \alpha] = 1 - \Pr[T_o \leq \Psi_v^{-1}(1 - p) \mid t_v, \alpha] = 1 - \Psi_{v, \alpha}^{-1}(1 - p),
$$

using the fact that under the alternative, the test statistic follows a non-central t-distribution with non-centrality parameter $\delta$ and $v$ degrees of freedom, and cumulative distribution and probability density functions $\Psi_{v, \alpha}(\cdot)$ and $\psi_{v, \alpha}(\cdot)$. Letting $\lambda_i$ be the probability that the $i$th voxel is truly active, and $\delta_i$ as the voxel-specific non-centrality parameter,

$$
\Pr(P_i \leq p; \lambda_i, \delta_i) = (1 - \lambda_i)p + \lambda_i\left[1 - \Psi_v^{-1}(1 - p)\right],
$$

from where it follows upon taking derivatives that the density of $P_i$ is

$$
f_{P_i}(p; \lambda_i, \delta_i) = (1 - \lambda_i) + \lambda_i\frac{\psi_v^{-1}(1 - p)}{\Psi_v^{-1}(1 - p)}, \quad 0 < p < 1.
$$

The density of the p-value at a voxel is thus a mixture of the standard uniform density and another density involving a parameter $\delta_i$. This density, illustrated for $v = 122$ and different values of $\lambda$ and $\delta$ in Figure 1, is used in our assessment methodology.

3. Methods

3.1. Imaging

All MR images were acquired on a GE 1.5 Tesla Signa system equipped with echo-planar gradients and using v5.8 software. Structural $T_1$-weighted images were obtained using a standard spin-echo sequence with TE/TR of 10/500 ms, and slice-positioning following the recommendations of Noll et al. (1997) to minimize intersession differences. For the fMRI
sessions, twenty-four 6 mm-thick slices parallel to the AC-PC line and with no gap between them were acquired using a single-shot spiral sequence (with TE/TR of 35/4000 ms) under a paradigm which involved eight cycles of a simple finger-thumb opposition task performed for 32s, followed by an equal period of rest, over 128 time-points. The paradigm was repeated for twelve separate sessions over a two-month period on a single volunteer after obtaining informed consent. All the paradigms were on the dominant right hand of the subject. Reconstructions were performed on an SGI Origin 200 workstation after transferring the data from the scanner. Automated image registration (AIR) was used to correct for motion-related artifacts in each replication, after which time series were generated at each voxel (Wood et al., 1998) and normalized to remove any linear drift in the data. Cross-session image registration among the twelve sessions was additionally performed to minimize any residual misregistration using the inter session registration algorithms of AFNI (Cox and Hyde, 1997). The default first image volume was taken to be the target against which the images were registered. Functional maps were created after computing voxel-wise t-statistics (and corresponding p-values) using a general linear model, discarding the first three image volumes (to account for T1 saturation effects) and assuming first-order autoregressive errors, using sinusoidal waveforms with lags of 8s. The choice of waveform represented the BOLD response, while the lag duration corresponded to when the actual BOLD response was seen from the theoretical start of the stimulus. Finally, a composite image cube of p-values of activation was created using the same procedure as above on the combined voxel-wise data from the twelve replications. (Activation maps as well as true activation and inactivation certainty measures were computed for this composite image cube and are reported in the Results section.) Since the goal of this experiment is to detect regions of activation that are positively associated with the right-hand index finger-thumb opposition motor task performed by the subject, the alternative is one-sided and hence the one-sided t-tests were used. The dataset was then transferred to a Dell Precision 650 workstation, having two 3.06GHz Intel(r) Xeon(tm) processors running the Fedora 10 2.6.27.9-159 Linux kernel, where all the algorithms and statistical analyses reported in this paper were performed using a combination of commands in the “C” programming language and the statistical software package R (2008) publicly available for download from www.R-project.org.

3.2. Statistical Methodology

Most statistical analyses of fMRI data involve fitting a (typically but not necessarily, general linear) model relating the observed time series at each voxel to the hemodynamic response function (HRF). A test statistic is then constructed and its corresponding p-value is obtained and used in identifying activation. In the development here, we use the fact that the t-test is commonly used in analyses; similar methodology can be developed for the Kolmogorov-Smirnov and other tests. Further, we use a one-sided t-test to illustrate and evaluate the methodology because our application uses a one-sided alternative: we can readily develop similar methodology for two-sided t-tests.

Let $M$ be the number of replications of the experiment. Let $P_i = \{p_{i,1}, p_{i,2}, \ldots, p_{i,M}\}$, where $p_{i,j}$ is the observed $p$-value of the one-sided $t$-statistic at the $i$th voxel and the $j$th replication. The likelihood function for the $i$th voxel is then given by

$$
\prod_{j=1}^{M} \left\{ (1 - A_i) + A_i \psi_{v_i} \psi_{v_j}^{-1} (1 - p_{i,j}) \right\}.
$$

The above model assumes that the true effect magnitude (captured in $A_i$) for each voxel does not vary over the replications. This assumption is similar to that made by the binomial models of Maitra et al. (2002) or Genovese et al. (1997). However, unlike the former, the model (6) incorporates voxel-specific prob-
abilities ($\lambda_i$) of true activation as well as non-centrality parameters ($\delta_i$). The degrees of freedom of the test statistic here depend on the replication, but can also be made voxel-specific, if needed. Under spatial independence, the likelihood for the entire set of voxels in all slices of the corrected image is the product of (6) over all voxels.

For each voxel, there are two parameters ($\lambda_i$ and $\delta_i$) to be estimated. Thus, if $N$ is the number of three-dimensional image voxels under consideration, there are $2N$ parameters that are to be estimated from the MN observed $p$-values using the likelihood model. In our estimation process, we assume that the observed $p$-values at each voxel are independent with any spatial relationships fully captured in the voxel-specific $\lambda$s and $\delta$s. Since these are voxel-specific and the observed $p$-values are independent, maximization can be separately done for each voxel. This has the benefit of speeding up computation, but has the limitation mentioned above, namely that the fixed effect magnitude does not vary over the replications. An alternative approach is to have replication-specific $\delta$s, rather than voxel-specific fixed effect magnitudes. Direct maximization would, however, be computationally impractical then: a possible recourse could be to the expectation-maximization algorithm of Dempster et al. (1977). We have not pursued this course in this paper.

We use Nelder-Mead’s downhill simplex method (Nelder and Mead, 1965) to find the ML parameter estimates (MLEs). Note that the likelihood model assumes that $\lambda_i$ is in the interval (0, 1) and that $\delta_i$ is positive so that the parameters are, in theory, identifiable from the likelihood function. That is, any two different values of ($\lambda_i$, $\delta_i$) in the parameter space give rise to different values of the likelihood function. Numerically, however, small values of $\delta_i$ make $\lambda_i$ unidentifiable, since the second component density function is then very close to unity, and any value of $\lambda_i$ yields essentially the same likelihood value. Figure 1 displays the density function for $\delta = 1, 2, 3$ as the mixing proportion $\lambda$ increases from 0.05 to 0.95. Note that when $\lambda = 0$ (not pictured), the density is standard uniform, thus is a horizontal line taking the value 1 for all $p$-values. This is quite distinct from any of the plotted functions in any of the plots in Figure 1. Therefore, we conclude that for $\delta_i > 1$, the two components in the mixture of (5) seem to be well-separated and identifiability in estimation does not appear to be a major issue. Consequently, we restrict $\delta_i > 1$ in our computations.

Once these parameter estimates are available, true activation and inactivation certainty measures of voxels identified as activated and inactivated can be computed. For let $\tau_i$ be the threshold at which the $i$th voxel is declared to be activated if it has a lower $p$-value. The thresholds are not necessarily voxel specific and can be assumed to be obtained by any method, such as those obtained by controlling the False Discovery Rate (Genovese et al., 2002) or the related methods surveyed in Nichols and Hayasaka (2003). However, we can also use methodology (see below) that maximizes the ML reliability efficient frontier (Maitra et al., 2002; Genovese et al., 1997) voxel-wise, which also follows from the methodology developed above. From the threshold values $\tau_i$s, we can compute certainty measures. To see this, note that the $i$th voxel would be identified as activated if its observed $p$-value $P_i$ is less than or equal to $\tau_i$. Then the true activation certainty ($\rho^+$) of this voxel is the posterior probability of a voxel being truly active given that it has been identified as activated. Using Bayes’ Theorem,

$$\rho_i^+ = \frac{\Pr(\text{ith voxel is truly active} \mid \text{ith voxel is identified as activated})}{\Pr(\text{ith voxel is truly active})}$$

$$= \frac{\Pr(\text{ith voxel is truly active and } P_i < \tau_i)}{\Pr(P_i < \tau_i)}$$

$$= \frac{\Pr(\text{ith voxel is truly active}) \Pr(P_i < \tau_i \mid \text{ith voxel is truly active})}{\Pr(P_i < \tau_i)}$$

$$= \frac{\lambda_i \left[ 1 - \Psi_{\nu,\delta} \Psi^{-1}_{\nu}(1 - \tau_i) \right]}{(1 - \lambda_i)\tau_i + \lambda_i \left[ 1 - \Psi_{\nu,\delta} \Psi^{-1}_{\nu}(1 - \tau_i) \right]}$$

(7)

where the numerator follows from (3) and the denominator from (4), directly.

In a similar spirit, the true inactivation certainty ($\rho^-$) of a voxel identified as inactive is defined to be the (posterior) probability that it is truly inactive when it has been correctly identified as so. Corresponding to the above, this can be obtained as

$$\rho_i^- = \frac{\Pr(\text{ith voxel is truly inactive} \mid \text{ith voxel is identified as inactive})}{\Pr(\text{ith voxel is truly inactive})}$$

$$= \frac{\Pr(\text{ith voxel is truly inactive and } P_i \geq \tau_i)}{\Pr(P_i \geq \tau_i)}$$

$$= \frac{\Pr(\text{ith voxel is truly inactive}) \Pr(P_i \geq \tau_i \mid \text{ith voxel is truly inactive})}{\Pr(P_i \geq \tau_i)}$$

$$= \frac{(1 - \lambda_i)(1 - \tau_i) + \lambda_i \left[ 1 - \Psi_{\nu,\delta} \Psi^{-1}_{\nu}(1 - \tau_i) \right]}{(1 - \lambda_i)(1 - \tau_i) + \lambda_i \left[ 1 - \Psi_{\nu,\delta} \Psi^{-1}_{\nu}(1 - \tau_i) \right]}$$

(8)

where the numerator and the denominator follow from the complement from unity of (2) and (4), respectively.

As indicated above, and although not a focus of this paper, the obtained parameter estimates can also be used with the model to obtain threshold values. To see this, let $\tau_i$ be the given threshold at the $i$th voxel. Then the probability of making a correct decision by thresholding the $i$th voxel at threshold $\tau_i$ is equal to

$$\Pr(\text{correct decision} \mid \text{truly inactive voxel}) \Pr(\text{truly inactive voxel})$$

$$+ \Pr(\text{correct decision} \mid \text{truly active voxel}) \Pr(\text{truly active voxel})$$

or equivalently,

$$(1 - \lambda_i)(1 - \tau_i) + \lambda_i \left[ 1 - \Psi_{\nu,\delta} \Psi^{-1}_{\nu}(1 - \tau_i) \right].$$

(9)

The above is also called the ML reliability efficient frontier (Maitra et al., 2002; Genovese et al., 1997), and given estimates for $\lambda_i$ and $\delta_i$, we can maximize the above probability of making a correct decision with respect to $\tau_i$ to get an optimal threshold. It may be noted that the optimal thresholds here are also voxel-specific, since the parameters $\lambda_i$ and $\delta_i$ are so. Hence, different voxels can be declared as activated at different threshold values, providing a data-driven approach to detecting activation after accounting for spatial context and other inhomogeneities that arise from the experiment.
4. Results

4.1. Variability in Activation

Figure 2 represents the observed $p$-values of activation for slices 7 through 22 in the first and last replications of the experiment. All displays reported in this paper are in radiologic views and overlaid on top of the corresponding $T_1$-weighted anatomical images. Note the large amount of variability in the observed $p$-values in between the two replications. In both cases, the region of the left primary motor cortex appears to be significantly activated in response to the task of right finger-thumb opposition. But, let us consider, for instance, slice 20 (bottom row, second image slice) which shows a substantially large area encompassing the primary left motor cortex with low $p$-values in the first replication (Figure 2a). There are other large areas also in this slice which have very low $p$-values. In Figure 2b however, the area with low $p$-values in this region is far more concentrated and primarily in the region of the primary left motor cortex.

These two figures illustrate the across-session variability in observed $p$-values for the same paradigm on the same subject. This variability can impact the results of experiments and scientific conclusions. To see this, consider the results of using the Benjamini and Hochberg (1995) approach to determining activation by controlling the expected false discovery rate (FDR) nominally at $q = 0.05$ separately, for each of the twelve experiments. Figure 3 displays radiologic views of the $p$-values of voxels determined as activated in the eighteenth through the twenty-first slices encompassing the ipsi- and contra-lateral pre-motor cortices (pre-M1), the primary motor cortex (M1), the pre-supplementary motor cortex (pre-SMA), and the supplementary motor cortex (SMA). Clearly, there is wide variability in the results. Thus, while all experiments identify activation in the left M1 and in the ipsi-lateral pre-M1 areas, there is wide variability in identified activation in the contra-lateral pre-M1, pre-SMA and SMA voxels, with some experiments (most notably, the fifth, eleventh and to a lesser extent, twelfth replications) reporting very localized or no activation, while in other cases, these areas are identified as activated and indeed, the identified activated regions are sometimes more diffused. Indeed, the $66 R_{js}$ range from 0.081 to 0.494, with a median value of 0.228 and an inter-quartile range of 0.115. Figure 3 illustrates the need for variability assessment very nicely. Conclusions based on any of the twelve replications that do not account for the variability in the experiment could be very different and potentially erroneous. Hence, some quantification of
Figure 3: Radiologic view maps for (a) slice 18, (b) slice 19, (c) slice 20 and (d) slice 21, of $p$-values for activation regions as determined by controlling the False Discovery Rate (FDR) at a nominal expected FDR of $q=0.05$. For each slice, we display the $p$-values of activation for the thresholded voxels using a $t$-test of the motor function for the twelve replications of the right hand finger-thumb opposition experiment on the same volunteer. Note the differences in location and extent of activation over the twelve replications.
variability in the observed activation is needed. We demonstrate use of our methodology towards this goal in the next section.

4.2. Illustration of Methodology

Our Nelder-Mead minimization routines for the converged ML estimates of $\lambda$s and $\delta$s at each voxel took around 5 milliseconds. Thus calculations on the estimated parameters for the entire set of images took a little more than half an hour. Figure 4 displays the estimated $\lambda$s and $\delta$s for slices 18, 19, 20 and 21. Note that we process and estimate parameters for all slices, but henceforth only display these four slices for clarity of presentation. The $\lambda$-values are voxel-wise estimates of probability of true activation and it is encouraging to note from Figure 4a that they are high in known regions of activation such as the left M1, the ipsi- and contra-lateral pre-M1 areas, and also moderately so in the pre-SMA and SMA areas. Similar trends are also reported for the estimated $\delta$s (see Figure 4b).

Unlike in the setup of Genovese et al. (1997) or Maitra et al. (2002), every voxel has an individual ROC curve. Alternatively, the probabilities of true positives and false negatives ($\pi_A$ and $\pi_I$) take different values for the same thresholds at different voxels. For instance, at the threshold $\tau_j$, $\pi_I$ is also $\tau_j$ regardless of voxel while from (3), we get $\pi_A | = 1 - \Psi^{\nu,\delta} \left[ \Psi^{\nu,\delta}(1 - \tau_j) \right]$ for the $i$th voxel. Figure 5 summarizes the ROC voxel-wise in terms of the area under the curve (AUC) for the four slices. The AUC is an average of the sensitivity over all possible specificities (Swets, 1979; Hanley and McNeil, 1982; Metz, 1986), with high values indicating good discrimination between truly activated and inactivated voxels. Thus, it is encouraging to note that the AUCs are very high in areas such as the left M1 that are

Figure 4: Estimated (a) $\lambda$ and (b) $\delta$ images for slices 18, 19, 20 and 21 (row-wise) in the right hand finger-thumb opposition experiment. A contour plot of anatomic detail of each slice is overlaid on the corresponding image.

Figure 5: Area under the voxel-wise estimated receiver operating characteristic (ROC) curves for slices 18, 19, 20 and 21 for the right hand finger-thumb opposition experiment.
known to have a high chance of true activation ($\lambda_i$), providing confidence in the results derived using our modeling approach.

Figure 6 displays the derived voxel-wise cutoffs optimizing the ML reliability efficient of (9) for the four slices. These are the thresholds at which our confidence in the reliability of detected activation is the greatest. The figures indicate that our confidence of a correct activation is highest for truly active voxels (such as left M1) at higher thresholds whereas very low thresholds are required for greater accuracy in other regions.

Comprehensive activation maps were obtained by thresholding the $\rho$-values of the composite image with the voxel-wise thresholds of Figure 6. These maps along with their $\rho^+$ and $\rho^-$ values are displayed in Figures 7. Note that the $\rho^-$-values for this figure are quite high, corresponding to a lowest value of 0.62. We also calculated the true certainty measures (Figure 7b) for the areas of activation and inactivation identified on the composite image by the FDR thresholding methods ($q = 0.05$) of Benjamini and Hochberg (1995) as adapted to fMR data by Genovese et al (2002). (Results obtained using the more liberal approach of Storey and Tibshirani (2003) were essentially the same as in Figure 7b and are not displayed.) These two figures very nicely illustrate the value of certainty assessment and also the performance of our method in the context of threshold-identification and certainty calculations. To see this, note that the areas identified as activated by FDR, such as the left ipsi-lateral pre-M1 and left M1 areas are a subset of those that are identified as activated using the thresholding of our method. This is encouraging because FDR methods are known to be overly conservative when a large number of null hypotheses are known to be true, as is the case in fMRI experiments where most voxels are known to exhibit no activation. At the same time, voxels in the SMA, pre-SMA and contra-lateral pre-M1 regions that are identified as activated using our thresholding but inactivated using FDR have high $\rho^+$ and $\rho^-$ values respectively. This is very encouraging because this means that the more conservative FDR method has missed areas of activation (such as in the pre-SMA, SMA, and contra-lateral pre-M1 regions) that our thresholding picked up with correspondingly high $\rho^+$-values but these have low $\rho^-$-values under the FDR thresholding. Thus even though these areas were not identified as activated by FDR, they have a good chance of being truly active, illustrating the value of assessing certainty in the results using our estimation method. Further, even though different thresholdings are used in the two approaches of Figures 7a and b, yielding different values for $\rho^+$ and $\rho^-$, the results are consistent. Thus, for any fMR experiment with test-retest data, we can not only obtain an activation map, but also a detailed map of the true activation and inactivation certainties of voxels that are identified as activated or inactivated, providing a tool for the investigator to quantify results.

### 4.3. Assessment of Methodology

The methodology was evaluated through a series of numerical experiments performed by generating $M$ replicated three-dimensional images of simulated $p$-values using the density in (5) and with the $\lambda_i$s and $\delta_i$s estimated from the above dataset as the “ground truth”. Our methodology was then used to estimate the parameters of the model. Estimation performance was assessed in terms of the Root Mean Squared Error (RMSE) of the estimated $\hat{\lambda}_i$s and $\hat{\delta}_i$s obtained using our methodology on the simulated data. Formally, RMSE ($\hat{\lambda}$) = $\sqrt{\sum_{i=1}^{N}(\hat{\lambda}_i - \lambda_i)^2/N}$ while RMSE ($\hat{\delta}$) = $\sqrt{\sum_{i=1}^{N}(\hat{\delta}_i - \delta_i)^2/N}$. Further, since the estimated parameters impact performance of our methodology together and through the density, we calculated the squared Hellinger distance between the two densities: $\int_0^1 [\sqrt{f_{R}(p; \hat{\lambda}_i, \hat{\delta}_i)} - \sqrt{f_{R}(p; \lambda_i, \delta_i)}]^2 dp$ where $f_{R}(\cdot)$ is as in (5).
Figure 7: Activation maps on the composite image from twelve replications obtained using the cutoffs derived from (a) maximizing the reliability efficient frontier and (b) the expected false discovery rate, along with their corresponding true activation (in red) and true inactivation certainties (in blue) for the right-hand finger-thumb opposition experiment.

We repeated the process for $M = 2, 3, \ldots, 12$ to assess how performance changes with different numbers of replications. Performance measures on the RMSEs and the averaged Hellinger distance over all the voxels are in Table 1. Note that while the RMSEs are modest for $\hat{\lambda}$s, they are somewhat higher for the $\hat{\delta}$s. However, the squared Hellinger distance averaged over all voxels is quite low, pointing to good performance of the methodology. We note, however, that performance with only two replications is not very good, and perhaps at least three replications are needed. Interestingly, it is a bit unclear whether the RMSEs go down consistently with increasing number of replications. This potentially points to some ill-posedness in the estimation process – a view that is further strengthened by noting that the RMSEs and squared Hellinger distance measures are heavily inflated by a few scattered voxels. Hence, incorporating some amount of regularization through a penalty function on the $\lambda$'s and the $\delta$'s as in Maitra et al. (2002) may be appropriate.

The robustness of the methodology in detecting activation and the certainty measures was also evaluated. The twelve replications in the dataset were randomly subdivided into two groups of six each, and our methodology was applied to each subset to obtain estimated parameters, as well as the thresholds maximizing the ML reliability efficient frontier. These were used separately to compute the $\rho^+$ and $\rho^-$ measures of the composite image map of $p$-values. Figure 8 shows these maps using the two random subsets of six replications each. It is encouraging to note that the activation maps as well as the true certainty measures are essentially the same for both Figures 8a and b, pointing to robustness of the suggested methodology in detecting activation.

5. Discussion

Genovese et al. (1997) and Maitra et al. (2002) provided novel approaches to estimating the test-retest certainty of a voxel using ML and its penalized version to enforce spatial dependence between the estimated parameters. In both cases, the approach needs some processing by thresholding the acquired fMRI data before the models can be applied. The number of threshold levels and the thresholding values are subjective and depend entirely on the investigator. This paper removes the need for this step by modeling the $p$-values of activation directly as a mixture of two distributions – one under the null hypothesis of no activation, and the other under the alternative hypothesis of true activation. Most fMRI data are processed using $t$-statistics obtained after fitting a general linear model, and we illustrate our methodology under this setup. We use this model and ML methodology to estimate the voxel-wise probability of true activation, and also other model parameters such as the non-centrality parameter which is allowed to be voxel-specific in order to account for systematic variations owing to local inhomogeneities in the magnetic field. These estimated parameter values can be easily used to obtain optimal thresholding values in order to determine if a voxel is activated or
inactivated. True activation and inactivation certainty measures of the activated and inactivated voxels can then be calculated and used by the investigator to obtain a quantitative assessment of the extent of activation. Voxel-specific ROC curves were also obtained for each voxel. Finally, the method was evaluated for its estimation performance and also for robustness in detecting and quantifying certainty of activation.

Two reviewers have very kindly asked about the practical utility of the derived methodology. This paper demonstrates certainty calculations on replicated single-subject experimental data. The end result is an individual activation map along with corresponding certainty measures of activation and inactivation. This provides understanding and quantitation of the activation of single-subject brains, which is important for clinical purposes. The methodology can also be used in the context of replicated and non-replicated data on the same experimental task or condition performed by multiple subjects. For each subject, one would draw an activation map and calculate the individual certainty measures of activation and inactivation. Once again, individual certainty measures for each subject could potentially be useful for clinical diagnosis: for instance one may be interested in finding out reasons for an individual’s low certainty measures of activation/inactivation in understanding how his brain compares with the rest. These measures can provide the researcher and the neurologist with a starting point for clinical investigation and diagnosis.

The certainty measures estimated in this paper were dependent entirely on the statistical analysis chosen to prepare the activation maps. Thus, it is imperative that fMR data are adequately cleaned and post-processed before analysis. For instance, one may have draining veins in an area as determined by an MR angiographic scan. In this case, an appropriate approach would be to mark the voxels in this region as inactive and use this additional information in the modeling and estimation. Other more sophisticated analysis such as in Saad et al. (2001) may also be considered. Further, data may also be digitally filtered (Genovese et al., 1997) prior to analysis in order to account for physiological factors such as cardiac and respiratory motion which greatly degrade the quality of activated maps.

There are a number of other remaining issues that merit further attention. In the derivations and analysis in this paper, we ignored any spatial structure among the parameter values in the estimation process. Approaches such as in Maitra et al. (2002) can be easily incorporated in the model and are a natural extension. This would also allow for incorporating smoothness that is introduced, as kindly suggested by a reviewer, in the registration step of pre-processing. This would also help in reducing the number of replications needed, and also in providing statistical consistency in the estimates, as mentioned in Section 4.3. Further, the methodology suggested in this paper was developed using $t$-tests. One advantage of the thresholding approach of Genovese et al. (1997) is that replications analyzed using different testing strategies could be analyzed together us-

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Figure 8: Activation maps on slices 18, 19, 20 and 21 (row-wise) of the composite image from twelve replications with corresponding true activation (red) and inactivation certainties (blue) obtained from parameters and thresholds estimated using (a) a randomly chosen sample of six replications from the data and (b) the other six replications, complementary to the set in (a). Displays are as in Figure 7.
ing very little additional effort. Though our entire development here used the most commonly used t-tests, our methodology is general enough to be modified and extended to situations involving other kinds of analysis (such as Kolmogorov-Smirnov tests), or when the replicates are analyzed using different testing strategies. In this case, the model underlying these other testing strategies will need to be explicitly incorporated in the development.

A separate issue involves applicability of this methodology to grouped fMRI data, such as in Gullapalli et al. (2005). It would be illustrative to see how certainty of activation/inactivation with grouped data using our suggested method compares with that done in that paper. One could also compare with the other traditional measures of reliability for grouped data, such as the ICC. Further investigations are also needed in order to test the utility of the methodology to studies done using other paradigms. Finally, one issue of great interest to researchers in cognitive sciences is to determine the certainty of activation maps obtained from a single-session study. In many cases, the nature of the experiment makes it impossible to have more than one session (hence replication) to acquire fMRI data. The test-retest methodology derived in this paper is inapplicable in such situations, and there is therefore great need for similar methods for such a scenario. One possibility is to model the runs, each of which occurs when a task is performed during a single fMRI visit or replication. Typically, multiple runs occur within the same replication. Note that runs necessarily have a dependence structure between them which will need to be modeled. Thus, while this paper introduces promising methodology to assessing certainty in test-retest fMRI activation studies, a number of issues remain that merit further attention.

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