IE 534 Linear Programming Homework Solution 8

1. Finish the proof of Proposition 5 in the lecture notes.

   Read the updated lecture notes for the complete proof.

2. Solve the following LP with the simplex algorithm and the method given in lecture notes point 31. All simplex dictionaries should be shown, but do not use big-M method.

$$\text{P1:}\quad \begin{array}{ll} \max & \zeta = \begin{bmatrix} 3 & -11 & 14 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top \\ \text{s.t.} & \begin{bmatrix} 7 & 7 & -12 \\ 13 & 12 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \le \begin{bmatrix} -2 \\ -16 \end{bmatrix} \\ & \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top \ge 0. \end{array}$$

$$\text{D1:}\quad \begin{array}{ll} \min & \zeta = \begin{bmatrix} -2 & -16 \end{bmatrix} \begin{bmatrix} y_1 & y_2 \end{bmatrix}^\top \\ \text{s.t.} & \begin{bmatrix} 7 & 13 \\ 7 & 12 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \ge \begin{bmatrix} 3 \\ -11 \\ 14 \end{bmatrix} \\ & \begin{bmatrix} y_1 & y_2 \end{bmatrix}^\top \ge 0. \end{array}$$

   Construct a new problem

$$\text{D2:}\quad \begin{array}{ll} \min & \zeta = \begin{bmatrix} -2 & -16 \end{bmatrix} \begin{bmatrix} y_1 & y_2 \end{bmatrix}^\top \\ \text{s.t.} & \begin{bmatrix} 7 & 13 \\ 7 & 12 \\ -12 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ & \begin{bmatrix} y_1 & y_2 \end{bmatrix}^\top \ge 0. \end{array} \quad \text{and its dual is}$$

$$\text{P2:}\quad \begin{array}{ll} \max & \zeta = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top \\ \text{s.t.} & \begin{bmatrix} 7 & 7 & -12 \\ 13 & 12 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \le \begin{bmatrix} -2 \\ -16 \end{bmatrix} \\ & \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top \ge 0. \end{array}$$

   After two simplex iterations, we find (D2) unbounded. This means (P2) and (P1) are infeasible.