(30 points) Consider a linear program with complementarity constraints:

\[
\begin{align*}
\min_{x,y} & \quad c^T x + d^T y \\
\text{s.t.} & \quad Ax + By \geq f \\
& \quad 0 \leq y \perp q + Nx + My \geq 0,
\end{align*}
\]

where \( c \in \mathbb{R}^{n \times 1}, d \in \mathbb{R}^{m \times 1}, f \in \mathbb{R}^{k \times 1}, q \in \mathbb{R}^{m \times 1}, A \in \mathbb{R}^{k \times n}, B \in \mathbb{R}^{k \times m}, N \in \mathbb{R}^{m \times n}, \) and \( M \in \mathbb{R}^{m \times m} \) are parameters and \( x \in \mathbb{R}^{n \times 1} \) and \( y \in \mathbb{R}^{m \times 1} \) are decision variables. This problem can be reformulated as the following mixed integer linear program (MILP) by introducing a large finite parameter \( \theta \) and a binary decision variable \( z \in \mathbb{B}^{m \times 1} \):

\[
\begin{align*}
\min_{x,y,z} & \quad c^T x + d^T y \\
\text{s.t.} & \quad Ax + By \geq f \\
& \quad Nx + My \geq -q \\
& \quad -Nx - My \geq q - \theta z \\
& \quad -y \geq -\theta(1 - z) \\
& \quad x \text{ free, } y \geq 0, z \in \mathbb{B}^{m \times 1}.
\end{align*}
\]

We want to apply Benders decomposition to the MILP (4)-(9). We assign the variable \( z \) to the master problem and variables \( x \) and \( y \) to the sub-problem. Answer the following six questions:

1. What is the master problem?
2. What is the sub-problem?

3. What is the dual of the sub-problem?
4. Assuming that \( z^* \) is the optimal solution from the master problem, how to obtain a feasibility cut?

5. Assuming that \( z^* \) is the optimal solution from the master problem, how to obtain an optimality cut?
6. Draw a diagram of the algorithm.