4.4.11. Suppose that $f$ and $g$ are differentiable on an open interval $I$ and that $a \in \mathbb{R}$ either belongs to $I$ or is an endpoint of $I$. Suppose further that $g$ and $g'$ are never zero on $I \setminus a$ and that

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

is of the form $0/0$. If there is an $M \in \mathbb{R}$ such that $|f'(x)/g'(x)| \leq M$ for all $x \in I \setminus \{a\}$, prove that $|f(x)/g(x)| \leq M$ for all $i \setminus \{a\}$.

Is this result true if the limit of $f(x)/g(x)$ is of the form $\infty/\infty$?