

Directions: Work all six questions. Hand in any scratch work that you want graded. Answers with no supporting explanation will receive no credit.

1. Find the general solution of the equation

$$\left(\frac{y}{x} + 6x\right) dx + (\ln x - 2) dy = 0, x > 0.$$

2. Solve the initial value problem

$$y' = \frac{e^{-x} - e^x}{3 + 4y}, \quad y(0) = 1.$$

3. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

4. Find the general solution of the equation

$$y^{(6)} - y'' = 0.$$

5. Find the general solution of the equation

$$y'' + 4y = 3 \sin 2t.$$

6. You are given that the  $2 \times 2$  matrix  $\mathbf{A}$  has eigenvalues 1 and  $-2$ , an eigenvector corresponding to 1 is

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

and an eigenvector corresponding to  $-2$  is

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix}.$$

Compute  $e^{\mathbf{A}t}$ .