Lieberman - Math 166H Practice Exam #3

Directions: One hundred points total, each problem is worth 25 points. Calculators are allowed, but all numerical answers must be exact. (For example $\sqrt{3}$ is not the same as 1.73205080756888.)

You may use the following series:

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n,$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!},$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!},$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots,$$

$$(1+x)^p = 1 + \sum_{n=1}^{\infty} \binom{p}{n} x^n.$$

1. Suppose that the series

$$\sum_{n=0}^{\infty} a_n (x-2)^n$$

converges at $x = -1$. Can you conclude that the series converges at $x = 4$? Be sure to justify your answer.

2. Find a power series (of the form $\sum_{n=0}^{\infty} a_n x^n$) that converges to $\frac{x}{1+x}$.

3. Find the Taylor polynomial of order three based at $a = 1$ for the function

$$f(x) = e^x.$$

4. Find the Maclaurin polynomial of order three for $(1+x)^{-1/2}$ and estimate the error $R_3(x)$ if $-0.5 < x < 0.5$.