Directions: One hundred points total, each problem is worth 25 points. Calculators are allowed, but all numerical answers must be exact. (For example \( \sqrt{3} \) is not the same as 1.73205080756888.)

You may use the following series:

\[
\begin{align*}
\frac{1}{1+x} &= \sum_{n=0}^{\infty} (-1)^n x^n, \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!}, \\
\sin x &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \\
\cos x &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \\
\tan^{-1} x &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots, \\
(1 + x)^p &= 1 + \sum_{n=1}^{\infty} \binom{p}{n} x^n.
\end{align*}
\]

1. Suppose that the series

\[\sum_{n=0}^{\infty} a_n (x - 2)^n\]

converges at \( x = -1 \). Can you conclude that the series converges at \( x = 4 \)? Be sure to justify your answer.

2. Find a power series (of the form \( \sum_{n=0}^{\infty} a_n x^n \)) that converges to \( \frac{x}{1+x} \).

3. Find the Taylor polynomial of order three based at \( a = 1 \) for the function

\[f(x) = e^x.\]

4. Find the Maclaurin polynomial of order three for \( (1 + x)^{-1/2} \) and estimate the error \( R_3(x) \) if \(-0.5 < x < 0.5 \).