

Directions:

Answer five questions well (you may answer all six but only the best 5 scores will be used). Show your work. All answers must be justified by computation or explanation. All proofs must be in Statement/Reason format. You may use your pre-approved theorem list. You may NOT use a calculator.

1. Let $\mathbf{v} = [-4, 0, 1, 1]$, $\mathbf{w} = [0, 1, 2, 1]$. Compute or determine not defined:
 a) $-2\mathbf{v} + 3\mathbf{w}$ b) cosine of the angle between \mathbf{v} and \mathbf{w}

2. Let $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}$. Compute or determine not defined:

- a) $A + A^{-1}$ b) $B B^T$

3. Find the general solution to the system

$$x_1 + 2x_3 + x_4 = 0$$

$$x_1 + 2x_3 = 1$$

$$2x_1 + x_2 + 3x_3 + 2x_4 = 2$$

4. Let A be an $n \times n$ matrix. Prove that $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$. Note that H5A (of which this is a part) may not be used to in this problem.

5. Let A and B be symmetric $n \times n$ matrices. Prove that ABA is symmetric.

6. For an invertible matrix A , define the matrix A^\blacklozenge by $A^\blacklozenge = (A^{-1})^T$. Prove that $(BC)^\blacklozenge = B^\blacklozenge C^\blacklozenge$ (for B, C invertible $n \times n$ matrices).