

Read all 6 questions before you begin. Choose 5 (do not do all 6; I will grade only the first 5 I see). Each question is worth 15 points. All answers must be justified by computation or explanation. All electronic devices except calculators must be turned off. Calculators cannot contain any user-created program or other stored material.

- Find one real root of  $x^3 - 5.1x - 5 = 0$
- Make a case for one of the following: i) Fermat and Descartes independently invented analytic geometry; ii) Fermat is primarily responsible for the invention of analytic geometry; ii) Descartes is primarily responsible for the invention of analytic geometry. Defend your point of view with facts.
- Use Cavalieri's principle to show that the volume of the ellipsoid  $\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} = 1$  is  $16\pi$ .
- Explain why Newton and Leibniz are considered the inventors of calculus. Also discuss possible alternatives and their justification.
- Summarize the contributions of Leonhart Euler and describe briefly when and where he worked.

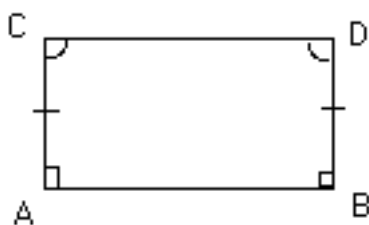
The following material has not been covered yet and so is not on Test 2 2008

- An isosceles bi-rectangle is shown in Figure 1 (angles A and B are right and sides AC and BD are congruent. It can be shown from Euclid's first 4 postulates that angles C and D are congruent, so you may assume that also). Euclid's 5<sup>th</sup> postulate is equivalent to the assumption

angles C and D are right angles (\*)

- State two alternatives to (\*) that when combined with Euclid's first 4 postulates (see below) result in two geometries (different from each other and each different from Euclid's).
- For each alternative you have started, discuss any necessary reinterpretation of Euclid's other four postulates. Reinterpretation means an interpretation different from Euclid's but still a possible interpretation of the postulate as stated.
- For each alternative you have started, state whether the sum of the angles in a quadrilateral is equal to, less than or greater than  $2\pi$ .

Figure 1



Euclid's Postulates

(<http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html>)

**Postulate 1.**

To draw a straight line from any point to any point.

**Postulate 2.**

To produce a finite straight line continuously in a straight line.

**Postulate 3.**

To describe a circle with any center and radius.

**Postulate 4.**

That all right angles equal one another.

**Postulate 5.**

That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.