Math 317 **Test 2** Oct. 20, 2005

Directions:
Answer Question 1 and three others well (you may answer all questions but only Question 1 and the best three of the others will be used). Show your work. All proofs must be in Statement/Reason format except showing a set is a subspace. You may use your pre-approved theorem list and may cite any correct result on it unless explicitly disallowed by the question. All answers must be justified by computation or explanation. You may use a calculator, and no work is required for reduction to reduced row echelon form (RREF), matrix inversion or for real arithmetic. For all other computations you must show work that justifies the answer without a calculator or be able to do it in your head.

1. (40 points, parts have differing weights) Determine whether the given subset W is a subspace of V (and show your answer is correct).

a) $W = \{p(x) \in P_2 \mid p(0) = 1\}$ $V = P_2$

b) $A = \begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$. $W = \{B \in R^{2x2} \mid AB = 0_{nxn}\}$ $V = \mathbf{R}^{2x2}$

c) $W = \{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x^2+y=0\}$ $V = \mathbf{R}^3$

2. (20 points) Find a basis for the subspace
$W = \{a-b-c+d +(a-d)x+(b+c-2d)x^2+(a-b)x^3]: a,b,c,d \in \mathbf{R}\}$ of P_3.

3. (20 points) Find the eigenvalues of $\begin{bmatrix} 2 & -1 & -1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix}$.

4. (20 points) Let A be a square matrix and suppose λ is an eigenvalue of A. Prove λ^2 is an eigenvalue of A^2.

5. (20 points) Prove H17: Let $A = [a_{ij}]$ be a lower triangular n x n matrix. Prove det A = $a_{11} \dots a_{nn}$. (Do NOT cite H17 for this problem).