Math 317 **Test 2** Oct. 22, 2002

Directions:
Answer four questions well, two from part 1 and two from part 2 (you may answer all questions but only the best 2 scores from each part will be used). Show your work. All proofs must be in Statement/Reason format except showing a set is/is not a subspace/linearly independent/spans W. You may use your pre-approved theorem list and may cite any correct result on it unless explicitly disallowed by the question. All answers must be justified by computation or explanation. You may use a calculator, and no work is required for reduction to reduced row echelon form (RREF) or for real arithmetic. For all other computations you must show work that justifies the answer without a calculator or be able to do it in your head.

PART 1

1. Determine whether the given subset W is a subspace of $\mathbf{R}^3$ (and show your answer is correct).

a) $W = \{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x+y=0\}$ b) $W = \{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid xy=0\}$

2. Find a basis for the vector space V.

a) $V = \{[r-s+t, r-s, r-s-t]: r,s,t, \in \mathbf{R}\}$ b) $V = \{p \in P_2: p(1) = 0\}$

3. a) Find the eigenvalues of $A=\begin{bmatrix} 2 & 3 \\ -1 & 6 \end{bmatrix}$. For each eigenvalue, find an eigenvector.

b) Let $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -5$. Compute $\det\begin{bmatrix} 8g & 8h & 8i \\ d+a & e+b & f+c \\ a & b & c \end{bmatrix}$

PART 2

4. Let A be a nonsingular matrix and suppose λ is an eigenvalue of A. Prove $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

5. Prove H19: Let $A_1,\ldots,A_k$ be n x n matrices. $\det(A_1\ldots A_k) = \det A_1 \ldots \det A_k$.(do NOT cite H19 for this problem).

6. Let V be a vector space and **v**, **w**, and **z** let be vectors in V. Prove that if a vector **x** is a linear combination of **v** and **w** then **x** is a linear combination of **v**, **w**, and **z** .(do NOT cite Theorem 4.6 for this problem).