

Table of selected optimal matrices

Laura DeLoss* Jason Grout* Leslie Hogben[†] Tracy McKay* Jason Smith*
Geoff Tims*

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Abstract

The minimum rank of a simple graph G is defined to be the smallest possible rank over all symmetric real matrices whose ij th entry (for $i \neq j$) is nonzero whenever $\{i, j\}$ is an edge in G and is zero otherwise. Minimum rank is a difficult parameter to compute. However, there are now a number of known reduction techniques and bounds that can be programmed on a computer; we have developed a program using the open-source mathematics software *Sage* to implement several techniques. We have also established several additional strategies for computation of minimum rank. These techniques have been used to determine the minimum ranks of all graphs of order 7. This report contains optimal matrices used to establish that the minimum rank of certain graphs is the lower bound found by the program.

Keywords. minimum rank, maximum nullity, zero forcing number, Sage program, mathematical software, symmetric matrix, rank, matrix, graph.

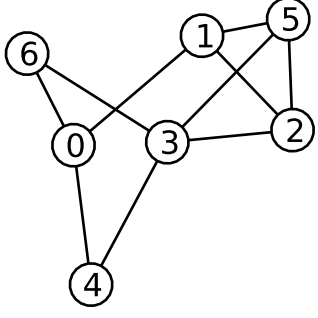
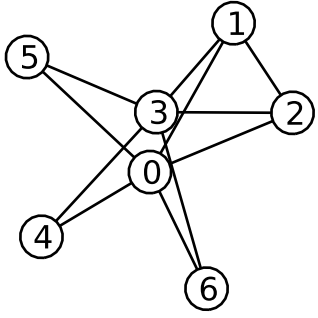
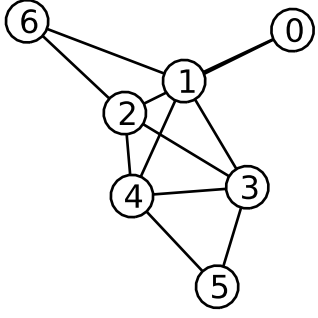
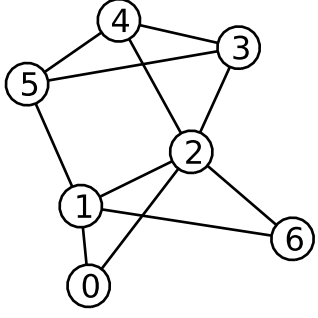
AMS subject classifications. 05C50, 15A03

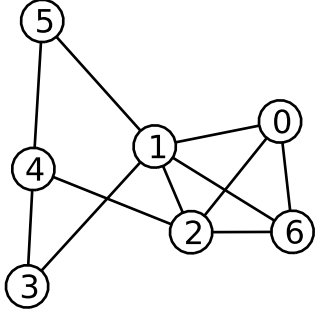
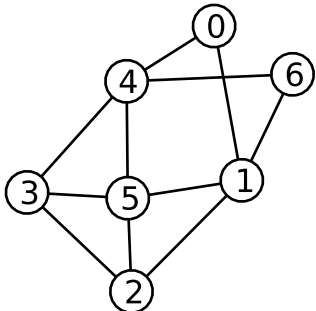
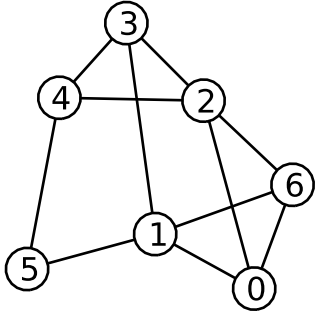
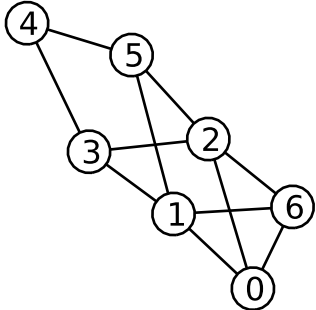
In this report, we list optimal matrices used to establish that the minimum rank of certain graphs is the lower bound found by the program. All these matrices have rank 3.

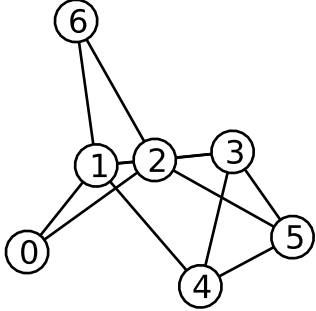
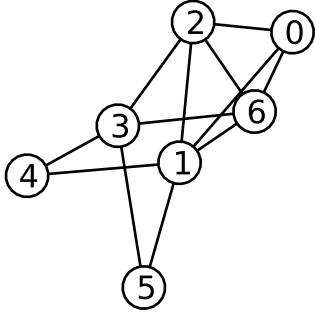
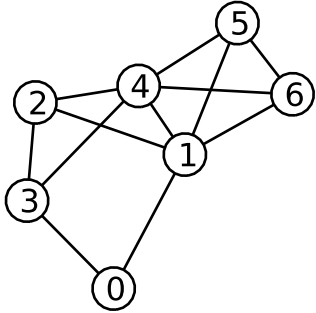
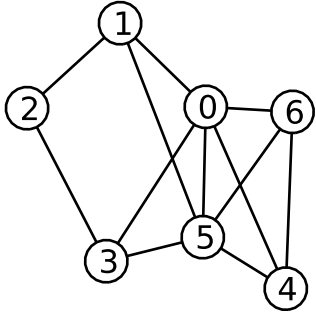
*Department of Mathematics, Iowa State University, Ames, IA 50011, USA (delolau@iastate.edu, grout@iastate.edu, tmckay16@iastate.edu, smithj@iastate.edu, gtims@iastate.edu).

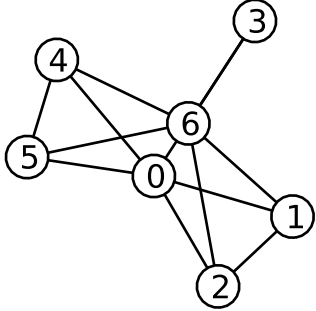
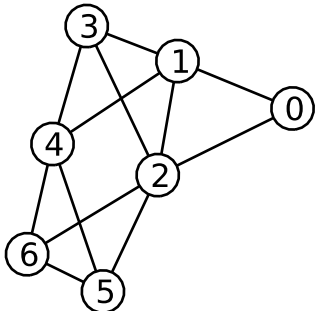
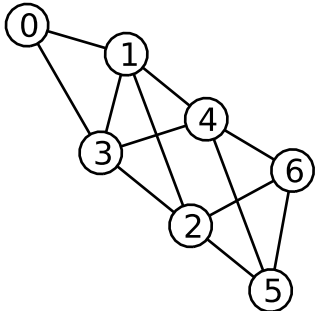
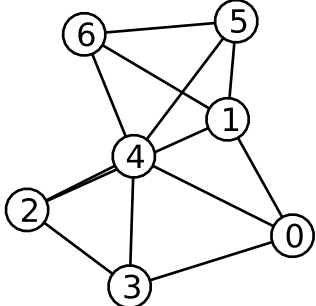
[†]Department of Mathematics, Iowa State University, Ames, IA 50011, USA (lhogben@iastate.edu) and American Institute of Mathematics, 360 Portage Ave, Palo Alto, CA 94306, USA (hogben@aimath.org).

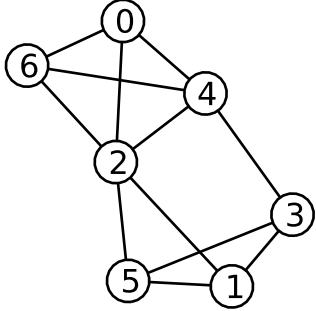
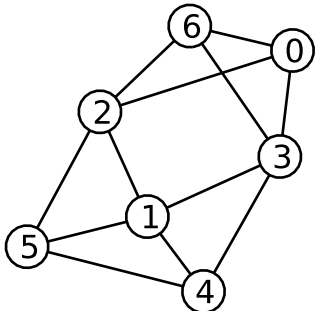
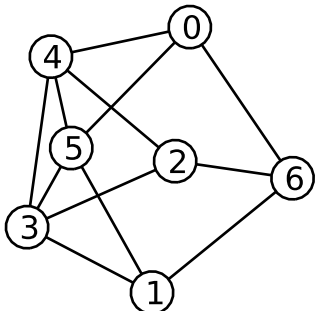
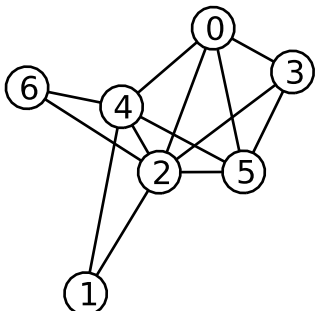
1 Selected optimal matrices

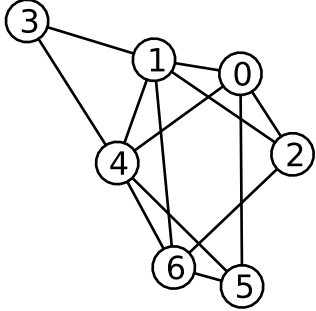
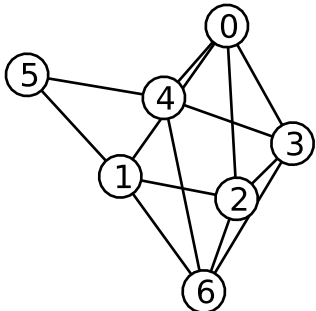
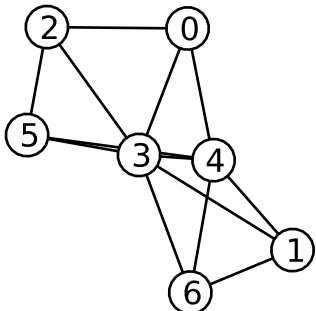
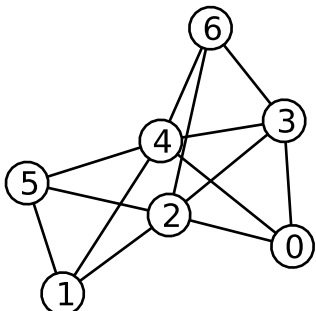
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801	$\begin{pmatrix} 0 & 1 & 1 & 0 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}$	
812	$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	
831	$\begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & -1 \\ 1 & 1 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$	

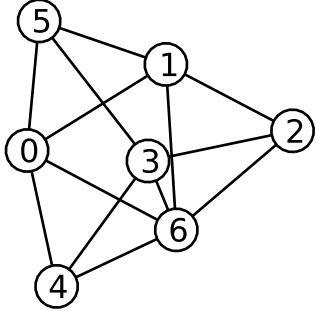
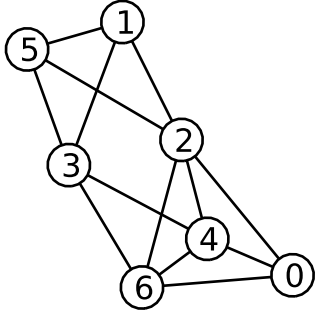
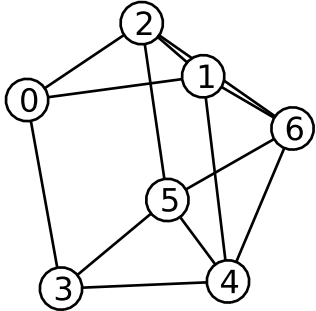
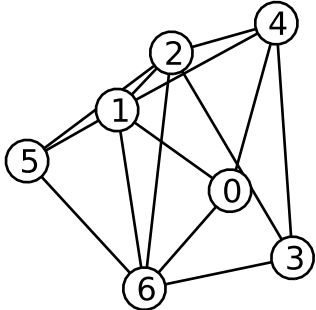
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846	$\begin{pmatrix} 0 & -2 & 0 & 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 1 & 2 & \frac{1}{2} & 2 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 & 0 \end{pmatrix}$	
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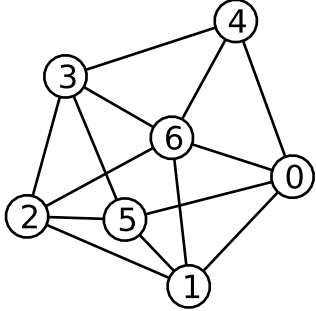
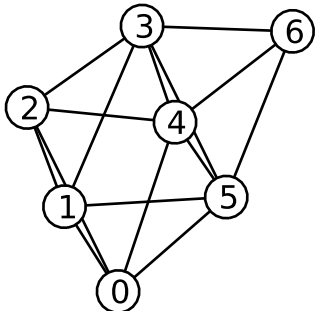
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956	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$	

958	$\begin{pmatrix} 0 & 1 & 1 & -2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ -2 & 0 & 0 & -2 & 0 & 0 & -2 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & -2 & 1 & 1 & 0 \end{pmatrix}$	
970	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & -1 & -1 & -1 \\ 0 & 0 & -1 & 0 & -1 & -1 & -1 \end{pmatrix}$	
990	$\begin{pmatrix} \frac{1}{8} & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 & 0 & 1 & 1 \\ \frac{1}{2} & 1 & 1 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$	
995	$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 2 & -1 & -2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$	

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1005	$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 2 & 1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & -1 & 1 & 2 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 & 3 \end{pmatrix}$	
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1060	$\begin{pmatrix} 2 & -1 & -1 & 0 & 1 & 1 & 0 \\ -1 & 2 & 1 & 1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 2 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 2 \end{pmatrix}$	
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1077	$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 2 & 1 & 1 & -1 \\ 1 & -1 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 1 \end{pmatrix}$	
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1095	$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 & -1 & -1 \\ 1 & 2 & 1 & 0 & 0 & -1 & 2 \\ 0 & 1 & 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 3 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & -1 \\ -1 & -1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 2 & 2 & 1 & -1 & 0 & 5 \end{pmatrix}$	
1099	$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 0 \\ 1 & 1 & 1 & 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & -1 & 0 & -1 \\ 0 & 2 & 1 & 1 & 0 & 2 & 0 \\ 1 & 0 & 2 & 1 & -1 & 0 & -1 \end{pmatrix}$	
1104	$\begin{pmatrix} -1 & 2 & 2 & 3 & 0 & 0 & 0 \\ 2 & -1 & 2 & 0 & 3 & 0 & 3 \\ 2 & 2 & -1 & 0 & 0 & 3 & -3 \\ 3 & 0 & 0 & -1 & 2 & 2 & 0 \\ 0 & 3 & 0 & 2 & -1 & 2 & -3 \\ 0 & 0 & 3 & 2 & 2 & -1 & 3 \\ 0 & 3 & -3 & 0 & -3 & 3 & -6 \end{pmatrix}$	
1146	$\begin{pmatrix} -1 & 1 & 0 & 0 & -1 & 0 & -3 \\ 1 & -2 & 1 & 0 & 1 & -1 & 4 \\ 0 & 1 & -2 & 1 & 1 & 1 & -4 \\ 0 & 0 & 1 & -1 & -1 & 0 & 3 \\ -1 & 1 & 1 & -1 & -2 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 & 1 \\ -3 & 4 & -4 & 3 & 0 & 1 & -19 \end{pmatrix}$	

1167	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 2 & 1 \\ 1 & -1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -1 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 1 & 1 & -1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & -1 & 1 & 1 & 0 & -1 & 0 \\ 1 & -1 & 2 & -1 & 1 & 0 & -2 \end{pmatrix}$	
1205	$\begin{pmatrix} \frac{3}{5} & \frac{1}{10} & -\frac{1}{2} & 0 & 1 & 1 & 0 \\ \frac{1}{10} & \frac{3}{5} & \frac{1}{2} & 1 & 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 3 & 1 & 1 & 3 \\ 1 & 0 & -1 & 1 & 3 & 1 & 3 \\ 1 & 1 & 0 & 1 & 1 & 3 & -1 \\ 0 & 0 & 0 & 3 & 3 & -1 & 7 \end{pmatrix}$	
1212	$\begin{pmatrix} 0 & 4 & -2 & 3 & 2 & 1 & 0 \\ 4 & -1 & 1 & 0 & 0 & 1 & 4 \\ -2 & 1 & -1 & 1 & 0 & 0 & -2 \\ 3 & 0 & 1 & -1 & 1 & 0 & 3 \\ 2 & 0 & 0 & 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 4 & -2 & 3 & 2 & 1 & 0 \end{pmatrix}$	