

colloquium

The Department of
Mathematics and ISU
ADVANCE welcome

**Susan
Montgomery**

Tuesday, September 2, 2008

Hospitality

3:45 p.m.

404 Carver Hall

Colloquium

4:10 to 5:00 p.m.

298 Carver Hall

Part of the
Women in
Mathematical
Sciences
Distinguished
Lecture Series

Orthogonal representations of Hopf algebras

In linear algebra, we say that an $n \times n$ matrix A is orthogonal if it preserves the usual inner product $(\ , \) : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$; that is,

$$(Av, Aw) = (v, w) \text{ for all } v, w \in V = \mathbb{R}^n.$$

More generally, for a group G and a representation V of G of dimension n over the complex numbers \mathbb{C} (that is, there is a group hom $\rho : G \rightarrow GL_n(V)$), we say that V (or ρ) is *orthogonal* for G if there exists a symmetric non-degenerate bilinear form $(\ , \) : V \times V \rightarrow \mathbb{C}$ such that

$$(g \cdot v, g \cdot w) = (v, w) \text{ for all } v, w \in V \text{ and } g \in G.$$

In fact in this case the representation is actually *real*: one can choose a basis $\{v_i\}$ for V such that $(v_i, v_j) \in \mathbb{R}$ for all i, j and $\rho : G \rightarrow GL_n(V')$, where V' is the span of the $\{v_i\}$ over \mathbb{R} . G is said to be *totally orthogonal* if all of its irreducible representations are orthogonal. Groups which are known to be totally orthogonal include all finite real reflection groups, for example the symmetric group S_n of degree n or the dihedral group D_{2n} of order $2n$.

Over \mathbb{C} , Frobenius and Schur gave a function $\nu(V)$ which predicts exactly when a given representation V is orthogonal; however in characteristic p one has to go back to the existence (or not) of a suitable bilinear form (for example in work of John Thompson).

Recently these notions have been extended to finite-dimensional Hopf algebras. In this talk, we review the classical results for the group algebra kG over a field k , and then discuss newer work concerning Hopf algebras constructed from finite groups, such as the Drinfel'd double $D(G)$ of a group G .

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