

November 29, 2010

Math 510 Ungraded Homework 14.1

Theorem 1. *Let $A \in F^{n \times n}$ have minimal polynomial $m_A(x) = (x - \mu_1) \dots (x - \mu_r)$ with $\mu_i \in F$ and $i \neq j$ implies $\mu_i \neq \mu_j$. Then*

1. $F^n = \ker(A - \mu_1 I) \oplus \dots \oplus \ker(A - \mu_r I)$.
2. *There exist $E_i \in F^{n \times n}, i = 1, \dots, r$ such that*

- (a) $E_i^2 = E_i$
- (b) $i \neq j$ implies $E_i E_j = 0$
- (c) $E_1 + \dots + E_r = I$
- (d) $\mu_1 E_1 + \dots + \mu_r E_r = A$
- (e) $\text{range } E_i = \ker(A - \mu_i I)$.

Most of this theorem was proved in slightly different form in September. Prove 2(e) is true. (Hint: The explicit construction of E_i used in the proof is useful.)