

November 19, 2010

Math 510 Ungraded Homework 13.3

1. Let P_2 be the inner product space of real polynomials of degree at most 2 with

$$\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx.$$

Let $f : P_2 \rightarrow \mathbb{R}$ be defined by $f(p) = p(1)$ (observe that f is a linear functional). Find a vector q such that for all $p \in P_2$,

$$f(p) = \langle p, q \rangle.$$

2. Let $A \in \mathbb{R}^{n \times n}$. Define $T_A : \mathbb{R}^n \rightarrow \mathbb{R}^n$ by $T_A(\mathbf{v}) = A\mathbf{v}$. What is $T_A^*(\mathbf{v})$ in \mathbb{R}^n with the dot product?
3. Let $B, H \in \mathbb{C}^{n \times n}$ with H positive definite. \mathbb{C}^n is an inner product space with $\langle \mathbf{v}, \mathbf{w} \rangle_H = \mathbf{w}^* H \mathbf{v}$. Define $T_B : \mathbb{C}^n \rightarrow \mathbb{C}^n$ by $T_B(\mathbf{v}) = B\mathbf{v}$. What is $T_B^*(\mathbf{v})$ in \mathbb{C}^n with the inner product $\langle \cdot, \cdot \rangle$?