

September 6, 2010

Math 510 Ungraded Homework 1.2

1. In \mathbb{R}^3 , let

$$\mathcal{B} = \left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right) \text{ and } \mathbf{v} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}.$$

Find $[\mathbf{v}]_{\mathcal{B}}$.

2. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by

$$T \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 5z - y \\ 3z \\ y + 2x \end{bmatrix}.$$

- (a) Find the standard matrix of T , $[T]$.
- (b) With \mathcal{B} as in #1, find the matrix of T with respect to \mathcal{B} , ${}_{\mathcal{B}}[T]_{\mathcal{B}}$.
3. A matrix $M \in F^{n \times n}$ is nilpotent if there exist a positive integer k such that $M^k = 0$. Let $A, B \in F^{n \times n}$. Prove: If AB is nilpotent then BA is nilpotent.