November 3, 2010

Math 510 Ungraded Homework 11.2

Let $A \in \mathbb{C}^{n\times n}$. Let $\beta_k, k = 1, \ldots, s$ be a collection of nonempty subsets of $\{1, \ldots, n\}$. We say the set of principal minors $A[\beta_1], A[\beta_2], \ldots A[\beta_s]$ is

- *nested* if $\beta_1 \subset \beta_2 \subset \cdots \subset \beta_s$ (where $\subset$ means proper subset).
- *complete* if for all $i = 1, \ldots, n$ there exists $k \in \{1, \ldots, s\}$ such that $|\beta_k| = i$.
- *positive* if $\det A[\beta_k] > 0$ for all $k = 1, \ldots, s$.

The canonical example of a complete set of nested principal minors is the leading principal minors, where $\beta_k = \{1, \ldots, k\}$ and $s = n$. Obviously a complete set of nested principal minors has n sets β_k, i.e., $s = n$.

1. Let $H \in \mathbb{C}^{n\times n}$ be Hermitian. Prove: If H has a complete set of nested positive principal minors, then H is positive definite.
2. Give an example of a Hermitian matrix that has a complete set of principal minors that are all positive but H is not positive definite (obviously the complete set of positive minors is not nested).