Math 590 Homework 3 Due Oct. 21, 2009

Directions: Concise, well-written mathematics is valued. There are only 4 problems.

1. Consider the perturbation matrix

$$A_\epsilon = \begin{bmatrix} 0 & 1 & & 0 \\ \vdots & \ddots & \ddots & \\ 0 & & & 1 \\ \epsilon & 0 & \cdots & 0 \end{bmatrix}$$

   of $A_0 \in \mathbb{C}^{n\times n}$. Find the eigenvalues and singular values of A_ϵ, $\epsilon > 0$. For small $\epsilon > 0$ (for example, let $n = 10$ and take $\epsilon = 10^{-10}$), compare the eigenvalues of A_ϵ and A_0. Do the same for the singular values.

2. Let $A \in \mathbb{C}^{n\times n}$. Show that $\text{rank A} = \text{rank A}^2$ if and only if the algebraic and geometric multiplicities of the eigenvalue $\lambda = 0$ are equal.

3. (3.2.18) If $A \in \mathbb{C}^{n\times n}$, then there exists unitary U such that $(A^*A)^{\frac{1}{2}} = UA$.

4. Table 1 gives ranks of $(A - \lambda I)^k$ for the eigenvalues λ of $A \in \mathbb{C}^{n\times n}$ and various k.

Table 1: $\mathrm{rank(A - \lambda I)^k}$

	$k=1$	$k=2$	$k=3$	$k=4$
$\lambda = 3$	8	7	6	6
$\lambda = -2+i$	9	8	8	8
$\lambda = -2-i$	9	8	8	8

   (a) What is n (the size of A)?

   (b) What is the characteristic polynomial of A?

   (c) Find the Jordan form of A.

   (d) Is it possible for A to be a real matrix? (Give a reason why or why not.)