1. Let $A, B \in \mathbb{C}^{n\times n}$.
   1. Suppose that A is invertible. Show that AB and BA are similar.
   2. Without using the theorem that AB and BA have the same nonzero eigenvalues, show that for any $A, B \in \mathbb{C}^{n\times n}$, AB and BA have the same characteristic polynomial.
   3. If both A and B are singular, must AB and BA be similar?

2. Give a one line proof of, or a counter-example to, each of the following statements, where $A, B \in \mathbb{C}^{n\times n}$:
   1. If A, B, C, D are all 1×1 matrices, then $\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(AD - CB)$.
   2. If A, B, C, D are all 2×2 matrices, then $\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(AD - CB)$.

3. Let $A = \begin{pmatrix} 1 & -2 & 5 & 0 & 11 \\ 0 & 3 & 0 & -1 & 4 \\ 0 & 0 & 3 & 8 & -8 \\ 0 & 0 & 0 & -5 & 5 \\ 0 & 0 & 0 & 0 & -5 \end{pmatrix}$.
   1. Determine the characteristic polynomial of A.
   2. State every possible minimum polynomial for any 5×5 complex matrix that has the characteristic polynomial you found in (a).
   3. Determine the minimum polynomial of A.