Math 510 Examination 1 Sept. 28, 2006

All answers must be justified by computation or explanation.

1. Let $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ where

   $$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 7 & -5 & 3 \\ 8 & -1 & 0 \\ 1 & 1 & 4 \end{bmatrix}.$$

   Compute the Schur complement of A in M.

2. Give a one line proof of, or a counter-example to, each of the following statements, where $A, B \in \mathbb{C}^{n\times n}$:

   (a) If A, B are both positive semidefinite, then $A + B$ is positive semidefinite.

   (b) If A, B are both Hermitian, then $A + B$ is Hermitian.

   (c) If A, B are both normal, then $A + B$ is normal.

3. Let $A \in \mathbb{C}^{n\times n}$ and $M = \begin{bmatrix} 0 & A \\ A^* & 0 \end{bmatrix}$. Show that the eigenvalues of M are $\sigma_1, \ldots, \sigma_n, -\sigma_1, \ldots, -\sigma_n$ where $\sigma_1, \ldots, \sigma_n$ are the singular values of A.

4. Let $A = \begin{bmatrix} 1 & -2 & 5 & 0 & 11 \\ 0 & 3 & 0 & -1 & 4 \\ 0 & 0 & 3 & 8 & -8 \\ 0 & 0 & 0 & -5 & 5 \\ 0 & 0 & 0 & 0 & -5 \end{bmatrix}$.

   (a) Determine the characteristic polynomial of A.

   (b) State every possible minimum polynomial for any 5×5 complex matrix that has the characteristic polynomial you found in (a).

   (c) Determine the minimum polynomial of A.