Math 510 | Homework 5 (corrected) | Due Monday, December 6

Directions: Concise, well-written mathematics is valued- one full page is (more than) sufficient for the solution to any one problem. There are only 4 problems (a Thanksgiving present). Software (for matrix arithmetic, algebra and calculus) may be useful for questions 1 and 4.

1. Exhibit a Householder transformation that maps $\begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$.

2. Let $A \in \mathbb{C}^{n \times n}$. Prove that the eigenvalues of $M = \begin{bmatrix} 0 & A \\ A^* & 0 \end{bmatrix}$ are $\pm\sigma_i$ where $\sigma_i, i = 1, \dots, n$ are the singular values of A.

3. Let $A \in \mathbb{C}^{n \times n}$ have $\sigma_1 = \sigma_n$. Prove A is a scalar multiple of a unitary matrix.

4. Let P_2 be the inner product space of real polynomials of degree at most 2 with

$$\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx.$$

Let $D : P_2 \to P_2$ be defined by $D(p) = p'$ (the derivative). Observe that D is a linear operator (you do not need to prove this). Determine $D^*(p)$ for $p(x) = ax^2 + bx + c$ where D^* is the adjoint of D.