Math 510 Homework 4 Due Wednesday, November 17

Directions: Concise, well-written mathematics is valued- one full page is (more than) sufficient for the solution to any one problem.

1. Give the requested example or a one sentence explanation why no such example exists. (Note: Assuming they are correct, very simple examples are acceptable.)
   a) A Hermitian matrix having Jordan Canonical form $\begin{bmatrix}\sqrt{2}\end{bmatrix} \oplus \begin{bmatrix}\sqrt{2}\end{bmatrix} \oplus \begin{bmatrix}-\sqrt{2}\end{bmatrix}$
   b) A normal matrix having Jordan Canonical form $\begin{bmatrix}\sqrt{2} & 1 \\ 0 & \sqrt{2}\end{bmatrix} \oplus \begin{bmatrix}\sqrt{2}\end{bmatrix}$
   c) A Hermitian matrix having Jordan Canonical form $\begin{bmatrix}1\end{bmatrix} \oplus \begin{bmatrix}-1\end{bmatrix} \oplus \begin{bmatrix}i\end{bmatrix}$
   d) A unitary matrix having Jordan Canonical form $\begin{bmatrix}1\end{bmatrix} \oplus \begin{bmatrix}-1\end{bmatrix} \oplus \begin{bmatrix}i\end{bmatrix}$

2. Let $A = UP$, where $A, U, P \in \mathbb{C}^{n\times n}$, U is unitary, and P is positive semdefinite (this is called a *polar decomposition* of A). Prove: A is normal if and only if $UP = PU$.

3. Prove: If H is a Hermitian matrix, and every principal minor of H is nonnegative, then H is positive semidefinite.

4. Prove: If H and K are $n \times n$ Hermitian matrices, then for $1 \le i \le n$
$$\lambda_i(H) + \lambda_1(K) \le \lambda_i(H+K) \le \lambda_i(H) + \lambda_n(K).$$

5. Prove the Theorem of the Alternative: For $A \in F^{m\times n}$ and $\mathbf{b} \in F^m$, either $\mathbf{b} \in \operatorname{range} A$ or there exists a vector $\mathbf{c} \in F^m$ such that $\mathbf{c}^T A = 0$ and $\mathbf{c}^T\mathbf{b} \neq 0$.