

Concise, well-written mathematics is valued. Each question is worth 2.5 points.

1. (cf. 4.1.7) Let $A, B \in F^{n \times n}$ be idempotent.
 - (a) Let $\text{char } F = 2$. Prove $A + B$ is idempotent if and only if $AB = BA$.
 - (b) Let $\text{char } F \neq 2$. Prove $A + B$ is idempotent if and only if $AB = BA = 0$.
2. (cf. 4.1.9) Let $A \in F^{n \times n}$.
Show that A is idempotent if and only if $(\text{rank}(A) = \text{tr}(A) \text{ and } \text{rank}(I - A) = \text{tr}(I - A))$.
3. (cf. 5.1.12) Let $A \in \mathbb{C}^{n \times n}$ be Hermitian. Show $(A - iI)^{-1}(A + iI)$ is unitary.
4. (cf. 7.1.17) Let $A \in \mathbb{C}^{n \times n}$ be Hermitian of rank $r > 0$.
Show A has a nonsingular principal submatrix of order r .
Is this true without the assumption that A is Hermitian? (Prove or give a counterexample.)