

Concise, well-written mathematics is valued.

1. (cf. 4.1.7) Let  $A, B \in F^{n \times n}$  be idempotent ( $\text{char } F \neq 2$ ).  
Find and prove (useful) necessary and sufficient conditions for  $A + B$  to be idempotent.
2. (cf. 4.1.9) Let  $A \in F^{n \times n}$ .  
Show that  $A$  is idempotent if and only if ( $\text{rank}(A) = \text{tr}(A)$  and  $\text{rank}(I - A) = \text{tr}(I - A)$ ).
3. (cf. 5.1.12) Let  $A \in \mathbb{C}^{n \times n}$  be Hermitian. Show  $(A - iI)^{-1}(A + iI)$  is unitary.
4. (cf. 7.1.17) Let  $A \in \mathbb{C}^{n \times n}$  be Hermitian of rank  $r > 0$ .  
Show  $A$  has a nonsingular principal submatrix of order  $r$ .  
Is this true without the assumption that  $A$  is Hermitian? (Prove or give a counterexample.)
5. (cf. 8.1.9) Let  $A \in \mathbb{C}^{n \times n}$ .  
Show that if  $A$  is normal and  $f(x) \in \mathbb{C}[x]$ , then  $f(A)$  is normal.  
Is it true that if  $f(A)$  is normal and  $f(x) \neq 0$  then  $A$  is normal? (Prove or give a counterexample.)