Math 510 Homework 3 (only change is due date) 9 A Monday October 25, 2010

Directions: Concise, well-written mathematics is valued- one full page is (more than) sufficient for the solution to any one problem.

1. Prove Theorem 4.1 in the Canonical Forms Notes, i.e., prove
   The characteristic and minimal polynomials of the companion matrix $C(f)$ are both equal to $f(x)$, i.e.,
   $$p_{C(f)}(x) = m_{C(f)}(x) = f(x).$$

2. Prove Theorem 5.7 in the Canonical Forms Notes, i.e., prove
   For $A \in F^{n\times n}$, the elementary divisors, and hence $\mathrm{RCF}_{ED}(A)$, can be found from the irreducible factors $h_i(x)$ of the characteristic polynomial $p_A(x)$ of A and the ranks of powers of $h_i(A)$. Specifically, the number of times $h_i(x)^k$ appears as an elementary divisor of A is
   $$\frac{1}{\deg h_i(x)}(\mathrm{rank}(h_i(A))^{k-1} + \mathrm{rank}(h_i(A))^{k+1} - 2\ \mathrm{rank}(h_i(A))^k).$$
   (You may assume A is similar to an RCF-ED matrix and of course it has been proved that A is similar to a Jordan matrix.)

3. Find $\mathrm{RCF}_{ED}(A)$ and $\mathrm{RCF}_{IF}(A)$ of A over $\mathbb{Q}$ if $p_A(x) = x^4(x^2+2)^4(x^3-5)$ and the ranks are as shown in Table 1.

Table 1: $\mathrm{rank}(h(A)^k)$

$k=$	1	2	3	4
$h(x)=x$	13	11	11	11
$h(x)=x^2+2$	11	9	7	7
$h(x)=x^3-5$	12	12	12	12

4. Let $A \in \mathbb{C}^{n\times n}$ be skew-Hermitian. Prove $\mathrm{spec}(A)$ is purely imaginary.

5. Prove that if $A \in \mathbb{C}^{n\times n}$ is an upper-triangular matrix such that $A^*A = AA^*$, then A is in fact a diagonal matrix.