Math 510 Homework 3 Due Oct. 6, 2006

Directions: Concise, well-written mathematics is valued. You may use any result covered in Ch. 1, Ch. 2, or Sections 3.1-3.3 of the text, covered in class through Sept. 29, and/or the fact that every square complex matrix is similar to a Jordan matrix (i.e., has a JCF).
For questions 1 and 2, the use of software such as Mathematica or Matlab is encouraged (please submit complete printout), and use of rational matrices is suggested for these problems, although decimal matrices are acceptable. The matrices for questions 1 and 2 are

$$A = \begin{bmatrix} \frac{7}{9} & \frac{7}{9} & -\frac{10}{9} \\ \frac{4}{9} & \frac{4}{9} & \frac{2}{9} \\ -\frac{13}{9} & -\frac{13}{9} & \frac{61}{9} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{13}{6} & -\frac{1}{6} & -\frac{5}{3} \\ \frac{7}{6} & \frac{11}{6} & -\frac{5}{3} \\ \frac{5}{6} & \frac{7}{6} & -\frac{7}{3} \\ \frac{5}{2} & \frac{1}{2} & -1 \end{bmatrix}.$$

1. Find a unitary matrix U and upper triangular matrix T such that $U^*AU = T$ for A given above. Show all steps.

2. Find a singular value decomposition of B for B given above.

3. (3.2.18) If $A \in \mathbb{C}^{n\times n}$, then there exists unitary U such that $(A^*A)^{\frac{1}{2}} = UA$.

4. Table 1 gives ranks of $(A - \lambda I)^k$ for the eigenvalues λ of $A \in \mathbb{C}^{n\times n}$ and various k.

Table 1: $\mathrm{rank}(\mathrm{A} - \lambda \mathrm{I})^{\mathrm{k}}$

	$k=1$	$k=2$	$k=3$	$k=4$
$\lambda = 3$	8	7	6	6
$\lambda = -2+i$	9	8	8	8
$\lambda = -2-i$	9	8	8	8

   (a) What is n (the size of A)?

   (b) What is the characteristic polynomial of A?

   (c) Find the Jordan form of A.

   (d) Is it possible for A to be a real matrix? (Give a reason why or why not.)

5. (3.4.19) If $A \in F^{n\times n}$, then $W = \{f(A) : f(x) \in F[x]\}$ is a subspace of the F vector space $F^{n\times n}$. Prove that $\dim W = \deg m_A(x)$ (where $m_A(x)$ is the minimal polynomial of A).