Math 510 Homework 2 Due 4:10 PM Friday Sept. 24, 2010

Directions: *Mathematica* or other software may be used for matrix arithmetic in solving Problems 4 and 5, but the method should be shown. A printout (with comments either typed or written on by hand) may be submitted as a solution. The *Mathematica* session on Monday Sept. 20 will provide information on doing matrix computations. Concise, well-written mathematics is valued- a solution to one of Problems 1, 2, or 3 should not exceed one full page.

1. Assume A, B, C, D are all square matrices of the same size, $DC = CD$, and $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$. Give a careful* proof that
$$\det M = \det(AD - BC).$$
* Do not assume the Block Determinant Theorem, i.e., $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(AD - CB)$. Do not say, "use a method of proof similar to the Block Determinant Theorem." Do either adapt the proof of the Block Determinant Theorem (giving all details) or give a different proof.

2. Let $A, B \in \mathbb{C}^{n \times n}$. Prove $p_B(A)$ is singular if and only if $p_A(B)$ is singular, where $p_M(x)$ is the characteristic polynomial of M.

3. Let $A \in F^{m \times n}, B \in F^{n \times p}$. Prove
$$\operatorname{rank}(AB) = \operatorname{rank} B - \dim(\operatorname{range} B \cap \ker A).$$

4. Let $A = \begin{pmatrix} -6 & -3 & 3 \\ 25 & 12 & -11 \\ 8 & 3 & -1 \end{pmatrix}$. Find an invertible matrix S such that $S^{-1}AS = T$ is upper triangular.

5. Let $T = \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$. Find an invertible matrix S such that $S^{-1}TS = T_1 \oplus T_2$ with T_1, T_2 upper triangular.