Math 510 Homework 2 Due Sept. 15, 2006

Directions: Concise, well-written mathematics is valued- your entire set of solutions should ideally fit on 2 (legible) pages and under no conditions should it exceed 3 pages of text material, although you may write each solution on a separate piece of paper if desired. You may use any result covered in Ch. 1 or Ch. 2 of the text or in class through Sept. 8. Vectors are **bold**.

1. Let $A \in \mathbb{C}^{m\times n}$. Prove $(I + A^*A)^{-1} = I - A^*(I + AA^*)^{-1}A$.
2. Let $A, B \in \mathbb{C}^{n\times n}$. Prove: If A is similar to B, then $S_k(A) = S_k(B)$ for all $k = 1, \ldots, n$.
3. Problem 2.6.3, i.e., adj(AB) = adj(B)adj(A).
4. Let $U, V \in \mathbb{C}^{n\times n}$ be unitary. Prove:
   (a) $|\det U| = 1$.
   (b) UV is unitary
5. Let $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a set of vectors in an inner product space V. The *Gram matrix* of S is the matrix $G = [g_{ij}]$ where $g_{ij} = \langle \mathbf{v}_i, \mathbf{v}_j \rangle$. Prove:
   (a) G is Hermitian.
   (b) G is positive semidefinite (i.e., if $\mathbf{w} \in \mathbb{C}^n$, then $\mathbf{w}^*G\mathbf{w} \geq 0$).
   (c) G is positive definite (i.e., if $\mathbf{w} \in \mathbb{C}^n, \mathbf{w} \neq \mathbf{0}$, then $\mathbf{w}^*G\mathbf{w} > 0$) if and only if S is linearly independent.