Math 510 Homework 1 Due Sept. 1, 2006

Directions: Because some of this homework covers undergraduate linear algebra that has not been carefully developed in this class, it may be unclear what results you can use. Thus I have listed an assortment at the end that I think is sufficient for problems marked with a *. If you have another result you want to use for a * problem, ask me by e-mail and I may allow it. For other problems, you may use any result covered in the text sections 1.1, 1.2, 2.2 or in class.

Vectors are **bold**.

1. * Without using any results about block matrices other than the definition, and using the Laplace expansion definition of the determinant (consistently on the row or column of your choice), prove that if A, D are square matrices over field F and $\begin{bmatrix} A & B \\ 0 & D \end{bmatrix}$ is a block matrix, then
$$\det \begin{bmatrix} A & B \\ 0 & D \end{bmatrix} = \det A \det D.$$

2. * Let $A \in \mathbb{C}^{n\times n}$. Prove: $\ker(\mathrm{A}^*\mathrm{A}) = \ker(\mathrm{A})$.

3. * Problem 15 in Section 1.1.

4. State clearly the effect of each type of generalized elementary row operation on the determinant of a block matrix on which the operation is performed (note: your statements may depend on the dimensions of the blocks). Give reasons for your answers (formal proof not necessary).

5. Problem 13 in Section 2.2.

Results that may be assumed:

- Any definition from the text unless the question specifies which one.
- High school algebra and properties of real and complex numbers.
- Anything in Section 1.1 except the problems.
- Basic arithmetic of matrices, including $A\mathbf{0} = \mathbf{0}$ and if $\mathcal{O}$ represents zero matrices of appropriate size, $\mathcal{O}A = \mathcal{O} = A\mathcal{O}$.
- Principle of induction.
- $(A + B)^* = A^* + B^*, (AB)^* = B^*A^*, (cA)^* = \bar{c}A^*$.
- $\mathbb{C}^n$ is an inner product space with $(\mathbf{v}, \mathbf{w}) = \mathbf{w}^*\mathbf{v}$.