Math 166 Test 1 Sept. 15, 2009

Directions:

The number of points allocated to each part is shown in parentheses. Show your work. No partial credit will be given for a wrong answer that does not show work; in some parts (where specified below), to obtain full credit work must be shown. Any numerical answer should be exact (e.g., π) or to 6 significant digits, (e.g., 3.14159).

This paper must be turned in with your answers, and removing it from the classroom constitutes academic misconduct. Calculators that are not communication devices are the only electronic devices permitted during this test (computers are prohibited). Use of an improper device or possession of a communication device that is turned on during an examination (regardless of use) is a violation of course policy and is grounds for immediate termination of your test with a grade of 0. A communications device is any electronic device capable of communicating over a distance of more than a few feet without physical connection, including but not limited to cell phone, pager, PDA, wireless computer, etc. This test is closed book, i.e., books, notes, etc. are prohibited.

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1. Let $f(x) = 2x^3 - 3x^2 + 1$. Let R be the region between $f(x)$ and the x-axis from $x = 0$ to $x = \frac{3}{2}$.

   (a) (5 points) Sketch the region R (show scale on axes and shade in the region).

   (b) (15 points, work must be shown to receive full credit) Set up the integral for the volume of the solid obtained by revolving region R about the y-axis.

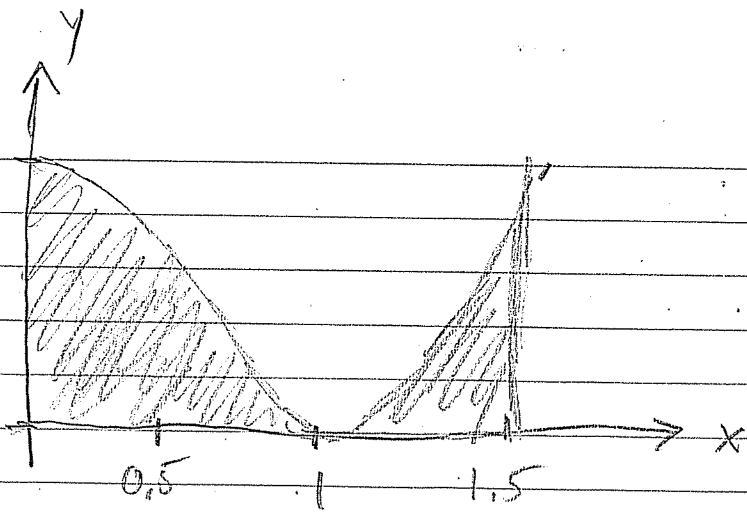
   (c) (5 points) Evaluate the integral you obtained in part (b).

   (d) (15 points, work must be shown to receive full credit) Set up the integral for the volume of the solid obtained by revolving region R about the line $y = 1$.

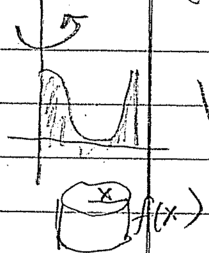
   (e) (5 points) Evaluate the integral you obtained in part (d).

2. (15 points, work must be shown to receive full credit) Let $f(x) = \cos(5x^2 + 2)$. Set up the integral for the arc length of the graph of $f(x)$ from $x = 0$ to $x = 3$.

1, (a)



(b) Use shells

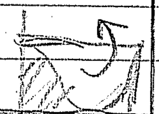


$$V = \int_a^b 2\pi x\, f(x)\, dx = \int_0^{3/2} 2\pi x\,(2x^3-3x^2+1)\, dx$$

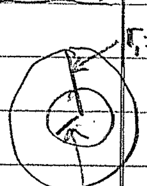
(c) $$= 2\pi \int_0^{3/2} 2x^4 - 3x^3 - x \, dx$$

$$= 2\pi \left(\frac{2}{5}x^5 - \frac{3}{4}x^4 - \frac{x^2}{2} \right) \Big|_0^{3/2}$$

$$= \frac{117\pi}{160} \approx 2.29729$$



(d) Use slices



$r_1 = 1$

$r_2 = 1 - f(x)$

$$V = \int_a^b \pi r_1^2 - \pi r_2^2 \, dx = \int_0^{3/2} \left[\pi \cdot 1 - \pi\left(1-(2x^3-3x^2+1)\right)^2\right] dx$$

(e) $$= \pi \int_0^{3/2} 1 - (3x^2-2x^3)^2 \, dx$$

$$= \pi \int_0^{3/2} 1 - 4x^6 + 12x^5 - 9x^4 \, dx = \pi\left(x - \frac{4}{7}x^7 + 2x^6 - \frac{9}{5}x^5\right)\Big|_0^{3/2}$$

$$\approx 951\pi/1120 \approx 2.66755$$

2 $f(x) = \cos(5x^2+2)$

$f'(x) = -\sin(5x^2+2)(10x)$

$L = \int_a^b \sqrt{1+f'(x)^2}\, dx$

$= \int_0^3 \sqrt{1+(10x\sin(5x^2+2))^2}\, dx$

Math 166 Test 1 Sept. 17, 2009

Directions:

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1. (15 points, work must be shown to receive full credit) Let $f(x) = \sqrt{x}\tan x$. Set up the integral for the arc length of the graph of $f(x)$ from $x = 0$ to $x = 1$.

2. Let $f(x) = -2x^3 + 3x^2 + 1$. Let R be the region between $f(x)$ and the line $y = 1$ from $x = 0$ to $x = \frac{3}{2}$.

   (a) (5 points) Sketch the region R (show scale on axes and shade in the region).

   (b) (15 points, work must be shown to receive full credit) Set up the integral for the volume of the solid obtained by revolving region R about the x-axis.

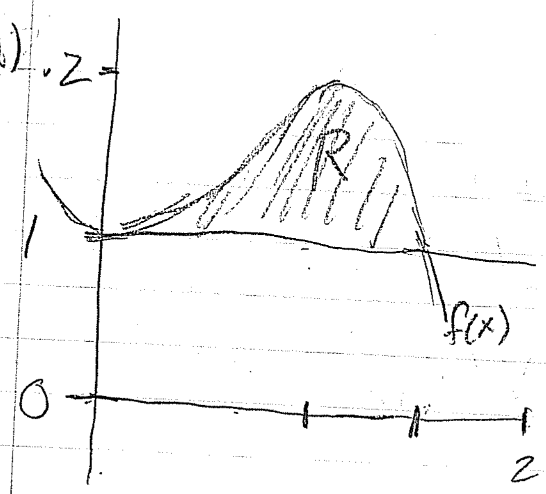
   (c) (5 points) Evaluate the integral you obtained in part (b).

   (d) (15 points, work must be shown to receive full credit) Set up the integral for the volume of the solid obtained by revolving region R about the line $x = -1$.

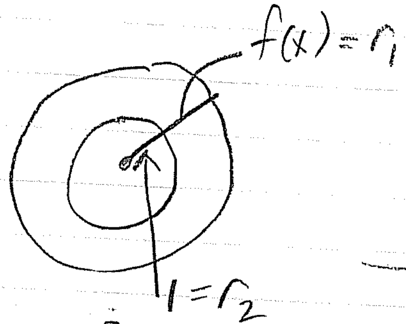
   (e) (5 points) Evaluate the integral you obtained in part (d).

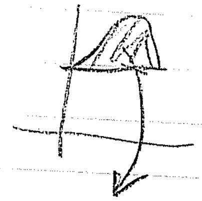
Math 166 Test 1 Thurs

2 (a)



b) By slices





$$V = \int_a^b \pi r_1^2 - \pi r_2^2 = \int_0^{3/2} \pi\left(-2x^3+3x^2+1\right)^2 - \pi(1)^2\,dx$$

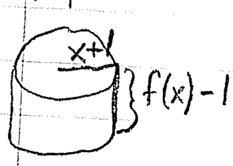
(c)

$$= \pi\int_0^{3/2} 6x^2 - 4x^3 + 9x^4 - 12x^5 + 4x^6\,dx = \pi\left(2x^3 - x^4 - \frac{9}{5}x^5 - 2x^6 + \frac{4}{7}x^7\right)\Big|_0^{3/2}$$

$$= 2619\pi/1120 \approx 7.34628$$

d) by shells

$$V = \int_0^{3/2} 2\pi\,(x+1)(f(x)-1)\,dx = 2\pi\int_0^{3/2}(x+1)(-2x^3+3x^2)\,dx$$



$$= 2\pi\int_0^{3/2} 3x^2 + x^3 - 2x^4\,dx = 2\pi\left(x^3 + \frac{x^4}{4} - \frac{2}{5}x^5\right)\Big|_0^{3/2}$$

$$= \frac{513\pi}{160} \approx 10.0727$$

1. $f(x) = \sqrt{x}\ \tan x$

$$f'(x) = \frac{1}{2\sqrt{x}} \tan x + \sqrt{x} \sec^2 x$$

$$L = \int_0^1 \sqrt{1 + f'(x)^2}\, dx = \int_0^1 \sqrt{1 + \left(\frac{1}{2\sqrt{x}} \tan x + \sqrt{x} \sec^2 x\right)^2}\, dx$$