Topic 9. Atomic and Nuclear Structure

MATTER AND ITS INTERACTIONS

Before discussing atomic and nuclear structure, let's briefly describe the physicist's current understanding of matter and the interactions between matter.

First, all matter (you, me, our books, our buildings, all animals and plants, all rocks and oceans and other planets and stars — and whatever is on them — as well as all matter, such as artificial elements, created in laboratories by physicists) consists of a few fundamental "matter" particles: the leptons and the quarks and their antiparticles.

You have probably learned that matter consists of ***molecules***, which themselves consist of ordinary ***atoms***, and that each atom consists of one or more ***electrons*** and a ***nucleus*** consisting of smaller particles called ***nucleons***. The nucleons include two types of particles, ***neutrons*** and ***protons***, each of which itself consists of three smaller particles called ***quarks***. It has taken chemists and physicists several centuries to discover all this, the quarks having been discovered in the last third of the 20th century.

The number of electrons in an atom equals the number of protons in its nucleus, and this number is called the ***atomic number Z*** of the atom. Each atomic number is associated with an element in the periodic table: $Z = 1$ for hydrogen (symbol: H), $Z = 2$ for helium (He), $Z = 3$ for lithium (Li), and so forth. The number of nucleons in the nucleus of an atom is called its ***atomic mass A*** (sometimes "atomic weight"). Since the number of protons is Z, the number of neutrons in the nucleus must be $A - Z$. Atoms that have the same value of Z are said to belong to the same element, while atoms that have the same value of Z but different values of A (meaning different numbers of neutrons) are said to be different ***isotopes*** of that element. The most common isotope of hydrogen is hydrogen-1, for which $Z = 1$ and $A = 1$, meaning this atom has 1 electron, 1 proton, and 0 neutrons. However, there are two other hydrogen isotopes in nature, hydrogen-2 (also called deuterium) and hydrogen-3 (also called tritium), differing only in the number of neutrons in their nuclei.

The three isotopes of hydrogen

Isotope	**Common name**	Z	A	**electrons** (Z)	**protons** (Z)	**neutrons** ($A - Z$)
Hydrogen-1	"ordinary hydrogen"	1	1	1	1	0
Hydrogen-2	deuterium	1	2	1	1	1
Hydrogen-3	tritium	1	3	1	1	2

At present the most fundamental particles of which we are aware, because as far as we know they are not made up of still smaller particles, are fundamental particles called ***leptons*** and ***quarks***. The electron is a lepton (and the only lepton encountered in ordinary, everyday matter), and the quarks are the constituents of the two types of nucleons. Each of these fundamental particles has associated with it certain definite physical properties, including mass and electric charge.

These two types of fundamental particles are usually called ***"matter particles"*** even though the term is not completely appropriate. There are six leptons, three of them having a negative electric charge and the other three zero electric charge, as well as their six antiparticles, which have the same masses as their corresponding particles, but opposite electric charges. The symbols for the leptons are shown in the table below, together with their names, and their electric charges in terms of the electron's electric charge, which is denoted $-e$.

Leptons & their electric charges

e^-	μ^-	τ^-
ν_e	ν_μ	ν_τ

$-e$
0

Names:

electron, muon, and tauon
electron's , muon's, and tauon's neutrino

Antileptons & their electric charges

e^+	μ^+	τ^+
$\overline{\nu}_e$	$\overline{\nu}_\mu$	$\overline{\nu}_\tau$

$+e$
0

Names:

antielectron (positron), antimuon, antitauon
electron's, muon's, and tauon's antineutrino

There are also six quarks, as well as their six antiquarks, shown in the table below along with their electric charges. The only quarks occurring in common everyday matter are the up and down quarks. The neutron consists of one up and two down quarks, and thus has zero electric charge, while the proton consists of two up and one down quarks, and thus has electric charge $+e$.

Quarks and their electric charges

u	c	t
d	s	b

$+2e/3$
$-e/3$

Names:

up quark, charmed quark, top quark
down quark, strange quark, bottom quark

Antiquarks & their electric charges

$\overline{u}$	$\overline{c}$	$\overline{t}$
$\overline{d}$	$\overline{s}$	$\overline{b}$

$-2e/3$
$+e/3$

Names:

up, charmed, and top antiquarks
down, strange, and bottom antiquarks

The quarks are unusual because they apparently do not occur singly in nature, but only in combinations of either three quarks ("baryons"), three antiquarks ("antibaryons"), or of a quark and an antiquark ("mesons"); these combinations – baryons, antibaryons, and mesons – are collectively referred to as "hadrons." (Neutrons and protons are the most common examples of baryons.) Normally, adding enough energy to any physical system will cause it to break up into its constituents. In the case of hadrons, however, adding energy leads instead to new hadrons, not single quarks.

In "ordinary" matter – naturally-existing matter as opposed to matter created in laboratories – only three different kinds of fundamental particles occur: the electron, the up quark, and the down quark. That's all! You and I and Brad Pitt and Angelina Jolie and the president and the queen and Saddam Hussein are just different combinations of those three fundamental particles, as are the rocks in the Grand Canyon, the lava from a volcano, the algae in the water pond, the Moon, the Sun – everything! As the French say, *Vive la difference!*

***Example 1*. The fundamental particles making up the water molecule**

Let's determine the makeup of the water molecule, including all the fundamental particles found in one molecule.

Ordinary water is made up of great numbers of molecules. Each molecule contains two hydrogen atoms and one oxygen atom, as its chemical formula - H_2O - indicates. What are the constituents of these three atoms? Each hydrogen atom (assuming it is the commonest hydrogen isotope, hydrogen-1) consists of one electron and one nucleus, and the nucleus consists of just one proton. The oxygen atom (again assuming it is the most common isotope found in nature, oxygen-16) consists of eight electrons (because $Z = 8$ for oxygen) and one nucleus, which has eight protons and eight neutrons. How many fundamental particles (leptons and quarks) are there in a single water molecule? There are 64 fundamental particles:

1. **Two hydrogen atoms**, each consisting of:
    a. one electron
    b. one nucleus, which consists of one proton, which consists of three quarks
2. **One oxygen atom**, which consists of
    a. eight electrons
    b. one nucleus, which consists of
        (i) eight protons, each of which consists of three quarks (total: 24 quarks)
        (ii) eight neutrons, each of which consists of three quarks (total of 24 quarks, but not the same three quarks as in the eight protons).

At the simplest level, each water molecule consists of **three atoms** (two hydrogen, one oxygen).

At the next level, each molecule consists of **10 electrons and three nuclei** (2 electrons in the two hydrogen atoms and 8 in the oxygen atom, plus two nuclei of hydrogen-1 and one of oxygen-16).

At the next level, each molecule consists of **10 electrons, 10 protons, and eight neutrons**.

At the most fundamental level, each molecule then consists of **10 electrons and 54 quarks** – a total of 64 fundamental particles!

Try to answer these review questions on your own, then check the answers given at the end of this section of the notes. If you got them right, congratulations! If you didn't, check to make sure you know how to answer them correctly.

Review Question 1. **Determine (in terms of A and Z) the number of leptons and quarks in one atom of an isotope with atomic number Z and atomic mass A.**

Review Question 2. **The two naturally-occurring isotopes of uranium ($Z = 92$) are uranium-235 and uranium-238.**

(a) Determine the number of electrons, protons, and neutrons in each isotope.

(b) Determine the number of leptons and quarks in one atom of each uranium isotope.

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(2) These matter particles are subject to a few fundamental interactions, each of which has its own field particles:

- ***The gravitational interaction (whose field particles are called gravitons)***
- ***The strong interaction (whose field particles are called gluons)***
- ***The electroweak interaction, which has two aspects:***
    electromagnetic (whose field particles are called photons)
    weak (whose field particles are the W and Z bosons)

The interactions (often referred to as "forces") that occur between matter particles can be classified into three interactions, each of which has one or more field particles associated with it. Every force that has ever been definitely observed in nature falls into one of the following three categories:

- **The gravitational interaction** responsible for the motions of the stars, planets, and satellites, as well as the tendency of objects to fall toward the earth. Its field particle is the **graviton**, a particle which has not yet been discovered, but which is associated with gravitational waves, indirect evidence for which has been found in certain binary star systems.

The original statement of a law for the gravitational interaction was Newton's Law of Universal Gravitation, that every particle in the universe attracts every other particle with a force that is proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them. The modern theory of gravitation was formulated by Albert Einstein in 1915 and called by him the General Theory of Relativity; it predicts certain subtle effects which have all been verified experimentally, but for most practical applications Newton's law is good enough. Newton's law of gravitation was introduced in Physics 111 and we will briefly review it at the beginning of Physics 112, so as to compare and contrast it with Coulomb's law of electricity.

- **The strong interaction** which holds together the particles found in atomic nuclei, including the binding of quarks into larger particles called hadrons and of hadrons into atomic nuclei. This interaction holds together the quarks in a proton, and it holds together the neutrons and protons in an atomic nucleus, even though the protons would normally repel one another electrically since they all have the same positive charge. The strong interaction is also responsible for nuclear fission and nuclear fusion, including the fusion processes in stars. The field particles of the strong interaction are the **gluons**, particles which do not exist freely in nature but which are present, along with quarks, in hadrons; there are a total of eight gluons, each of which has an antiparticle among the eight. We will only briefly discuss the strong interaction late in Physics 112.

- **The electroweak interaction**, which has two aspects referred to as the electromagnetic and weak interactions:

(1) The ***electromagnetic interaction***, which holds together the constituents of atoms and which is responsible for electrical and magnetic effects, including electromagnetic waves such as light, radio waves, and x rays. Its field particle is the **photon**, the massless quantum of the electromagnetic field. The basic equations of electromagnetism are Maxwell's Equations. Most of Physics 112 – all except the last few weeks – will deal with the electromagnetic interaction.

(2) The ***weak interaction***, which is responsible for the beta decay of free neutrons and of atomic nuclei. Its field particles are the **$W^{\pm}$ and Z^{0} bosons** discovered in 1983−84. The field particles of the weak interaction may be regarded as identical to the field particle of the electromagnetic interaction, the photon, except that they have mass and, in the case of the W particles, electric charge. When they have very high kinetic energies all these particles − photons, $W^{\pm}$, and Z^{0} − appear to be identical (except for electric charge). The massiveness of the weak interaction field particles is responsible for the fact that the weak force is a short-range force. This interaction will be briefly discussed the last week of Physics 112, when we discuss radioactivity.

The electromagnetic and weak interactions were initially regarded as distinct, but in the late 20th century the theories for these two interactions were unified into a single theory (just as, around 1860, Maxwell unified electric and magnetic interactions into a single theory of electromagnetic interactions). Physicists are currently trying to accomplish the next step, unifying the electroweak and strong interactions into a single "Grand Unified Theory" or GUT. Then, if they can unify all these forces with gravitation, they will have a "Theory of Everything" or TOE.

Solution to Review Question 1:

The number of leptons is the number of electrons, Z.

The number of quarks is three times the number of nucleons (protons plus neutrons), or $3A$.

The number of fundamental particles is then $Z + 3A$.

Solution to Review Question 2:

(a) Uranium-235: $Z = 92$, $A = 235$, $A - Z = 143$

92 electrons   92 protons   143 neutrons

Uranium-238: $Z = 92$, $A = 238$, $A - Z = 146$

92 electrons   92 protons   146 neutrons

(b) Uranium-235: $Z = 92$, $A = 235$

92 leptons $3 \times 235 = 705$ quarks

Uranium-238: $Z = 92$, $A = 238$

92 leptons $3 \times 238 = 714$ quarks

QUESTIONS

Consider the carbon dioxide molecule. The symbol for this molecule is CO_2, indicating that the molecule has one carbon (C) atom and two oxygen (O) atoms. Check a periodic table and find the atomic number Z for each of these atoms.

1. Z for carbon is: (1) 4 (2) 5 (3) 6 (4) 7 (5) 8
2. Z for oxygen is: (1) 4 (2) 5 (3) 6 (4) 7 (5) 8

The most common isotope of carbon has atomic mass 12 and that for oxygen has atomic mass 16.

3. The number of electrons in a neutral carbon atom is: (1) 4 (2) 6 (3) 8 (4) 10 (5) 12
4. The number of protons in the carbon nucleus is: (1) 4 (2) 6 (3) 8 (4) 10 (5) 12
5. The number of neutrons in the carbon nucleus is: (1) 4 (2) 6 (3) 8 (4) 10 (5) 12
6. The number of quarks in the carbon nucleus is: (1) 8 (2) 12 (3) 18 (4) 36 (5) 54
7. The number of electrons in a neutral oxygen atom is: (1) 6 (2) 8 (3) 10 (4) 12 (5) 14
8. The number of protons in the oxygen nucleus is: (1) 8 (2) 10 (3) 12 (4) 14 (5) 16
9. The number of neutrons in the oxygen nucleus is: (1) 8 (2) 10 (3) 12 (4) 14 (5) 16
10. The number of quarks in the oxygen nucleus is: (1) 16 (2) 24 (3) 36 (4) 48 (5) 72
11. The total number of electrons in the CO_2 molecule is: (1) 8 (2) 12 (3) 16 (4) 22 (5) 26
12. The total number of protons in the CO_2 molecule is: (1) 8 (2) 12 (3) 16 (4) 22 (5) 26
13. The total number of neutrons in the CO_2 molecule is: (1) 8 (2) 12 (3) 16 (4) 22 (5) 26
14. The number of quarks in the CO_2 molecule is: (1) 44 (2) 66 (3) 132 (4) 198 (5) 330

A less-common isotope of carbon has atomic mass 14; this is the one used in radiocarbon dating.

15. The number of electrons in a neutral C-14 atom is: (1) 4 (2) 6 (3) 8 (4) 10 (5) 12
16. The number of protons in the C-14 nucleus is: (1) 4 (2) 6 (3) 8 (4) 10 (5) 12
17. The number of neutrons in the C-14 nucleus is: (1) 4 (2) 6 (3) 8 (4) 10 (5) 12

A common ion of oxygen-16 isotope has charge -2, meaning that it has charge $-2e$ from having gained two electrons. The symbol for this ion of this isotope of oxygen can be written ${}^{16}_{8}\mathrm{O}^{-2}$.

18. The number of electrons in this ion is: (1) 4 (2) 6 (3) 8 (4) 10 (5) 12
19. How do the numbers of protons and neutrons in the nucleus of this ion differ, if any, from the numbers in the neutral atom of this isotope?

   (1) Two more protons but the same number of neutrons
   (2) Two more neutrons but the same number of protons
   (3) Two more protons and two more neutrons
   (4) The same number of protons and neutrons

PHOTONS AND ATOMIC SPECTRA

Now let's introduce atomic structure by beginning with this question: How are the e-m waves emitted by atoms related to energy levels of electrons?

Around 1900 it began to become clear that the principles of electromagnetism as understood at that time were not adequate to explain all observable phenomena. In particular, in order to obtain consistent explanations of the spectrum of e-m waves emitted by hot substances, some very unexpected assumptions were required. According to Maxwell's theory, e-m waves could have any desired energy, high or low (at least in principle). You could simply reduce the magnitude of the electric and magnetic fields as low as necessary in order to obtain e-m waves with energies as low as you desired. To do this, you could for instance produce very small amplitude oscillations of the charges that emitted the waves.

However, in 1900 German physicist Max Planck had proposed that the energies of these oscillators could ***not*** be as small as desired. Instead, they behaved as if there were a lower limit to their energies, and the value of this lower limit depended on the ***frequency*** of the oscillation according to this equation:

$$\Rightarrow \qquad \boldsymbol{E = hf},$$

where f was the frequency of the oscillator and h was a constant that later became known as "Planck's constant." ($h = 6.63 \times 10^{-34}$ J-s.) This appeared to be the "minimum package size" for the energy of the oscillating charges. In 1905, Einstein proposed that the e-m waves themselves could only be produced with energies of this "minimum package size." For instance, the lowest possible energy with which one could observe an e-m wave of 1000 Hz was

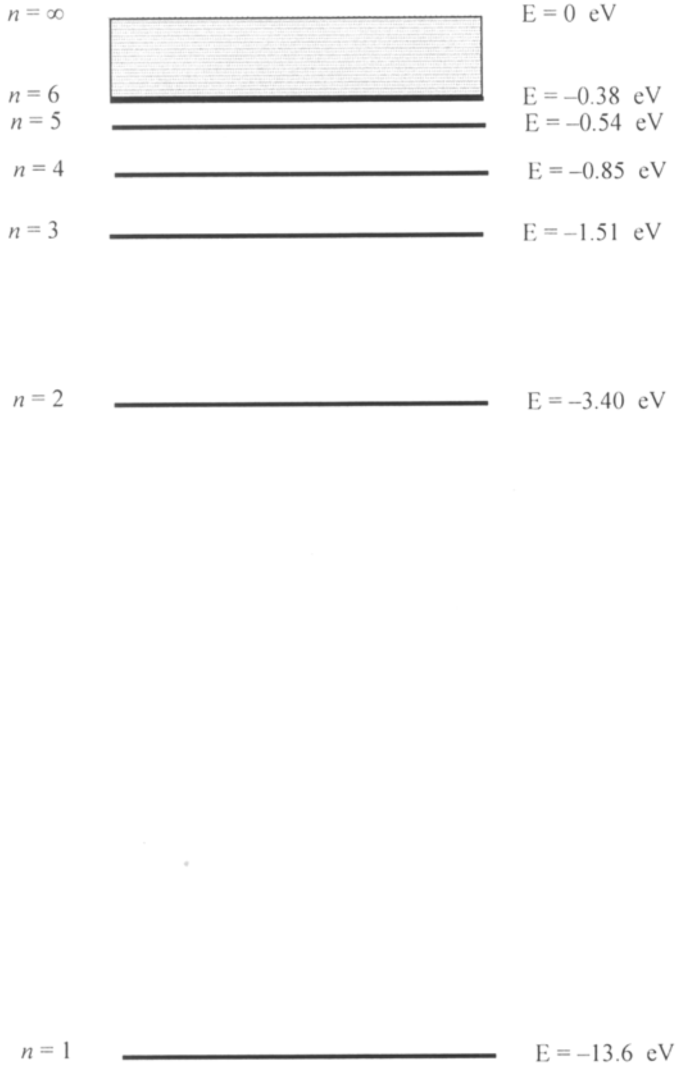
$$E = hf = (6.63 \times 10^{-34} \text{ J-s}) \times (1000 \text{ Hz}) = 6.63 \times 10^{-31} \text{ J}.$$

This is a very tiny amount of energy, but it was still very shocking that there might be any sort of a minimum energy value. This "minimum package" of e-m waves came to be called a "***photon***." So, the energy of a photon of e-m waves of a particular frequency could be found from the equation $E = hf$. High-frequency (short-wavelength) light has higher-energy photons than low-frequency (long-wavelength) light.

Around this same time, there was an intensive effort underway to understand the ***spectra*** of e-m waves emitted by gas atoms that had been struck by fast-moving electrons. It had been found that if one filled a glass tube with a low-pressure gas of any type, and then caused high-velocity electrons to race through the tube (by imposing a strong electric field on the tube), the gases would emit very distinctive "spectra" of e-m waves with various frequencies. These could be observed by a "spectroscope," which is a fancy version of a prism that turns white light into a rainbow. When the light emitted by a gas is passed through a spectroscope, a series of brightly colored lines is observed, with a different pattern of lines – different numbers of lines of different colors – characteristic of each different type of gas atom. These "emission spectra" are so distinctive they are a sort of fingerprint, which can be used to positively identify practically any substance (in gaseous form) by the spectrum of emitted e-m waves. Before 1913, there was essentially no understanding of how these spectra came to be produced.

At this time, the Danish physicist Niels Bohr proposed a model of atomic structure which provided some important clues as to how this process might work. Over the next 12 years his model underwent radical revisions by many physicists, but some of the basic mechanisms he proposed are still considered to be reasonably accurate. Bohr proposed that the electrons in any given atom could not, as previously believed, have any desired amount of energy. In a manner consistent with the ideas of Planck and Einstein, Bohr suggested that in each atom the electrons could only exist in a very limited and specific set of different "energy levels." This simply meant that the electrons could have those values of energy, and no others. Why this should be the case was not understood at that time. (Later on, the theory of quantum mechanics provides explanations for this phenomenon, producing very consistent results.)

Bohr considered the hydrogen atom in detail; with only one electron, hydrogen has the simplest structure of all atoms. By building on the observations and theories of other workers, Bohr came to propose that the electron in hydrogen could only take on a specific set of different energy values, of which the first six may be represented by the "energy level diagram" shown below.



In this diagram we have represented some allowed energy levels of the electron in a hydrogen atom. Ordinarily, the electron is in the "ground state" (corresponding to $n = 1$) and has energy of -13.6 eV. (1 eV = 1.6×10^{-19} J.) This means that you would have to add 13.6 eV of energy to the electron in order to break it away from the hydrogen atom. (If the electron just barely escaped from the atom – and had no energy left over to go anywhere – we would say that it has "zero" total energy, or $E = 0$ eV.) If an electron in the ground state ($n = 1$) gains energy its energy level would rise, for instance to $n = 2$, $n = 3$, or some higher value of n. The larger the value of n, the greater is the energy of the electron. The maximum possible energy for an electron in the hydrogen atom ($E \approx 0$ eV) would correspond to $n = \infty$.

With this model of the energy levels in hydrogen, the origin of the bright lines in the emission spectrum of hydrogen gas could now be explained. This is how it works: when fast-moving electrons are sent flowing through a tube of hydrogen gas, some of them collide with the hydrogen atoms and increase the energy of the electrons in the atoms. In this way, the hydrogen electrons may briefly acquire some of the higher energy levels shown in this diagram. Usually, after a brief moment, these electrons will abruptly lose at least some of their extra energy. When they do this, we say they "drop" in energy level. In this process, they can emit one single electromagnetic photon. The energy of the emitted photon must exactly equal the amount of energy that is lost by the electron as it changes its energy level.

When observing hydrogen gas in an electrical discharge tube (in which high-energy electrons are passed through the gas), the hydrogen glows with a colorful light. When the light is examined through a specially designed filter (a spectroscopic "grating"), it is possible to identify the wavelengths of the individual electromagnetic waves that make up that glowing light. Included among these is an e-m wave with a wavelength of 656.3 nm (1 nm = 1×10^{-9} m). We can identify the electronic transition in the hydrogen atom that corresponds to this e-m photon. (Here we'll need to recall that the energy of a photon of frequency f is given by $E = hf$; Planck's constant h has the value $h = 6.63 \times 10^{-34}$ J-s = 4.14×10^{-15} eV-s.)

We'll use the notation ΔE_{12} to represent the absolute value of $(E_{n=2} - E_{n=1})$; this is the energy change of an electron that makes a transition between level 1 and level 2. Then, according to Planck's formula,

$\Delta E_{12}/h =$ [frequency of e-m wave emitted when electron drops from $n = 2$ to $n = 1$].

The question is, how does the energy level n of the electron have to change in order to emit light with a wavelength of 656.3 nm?

Now, the frequency of light with a wavelength of 656.3 nm can be found from the relationship

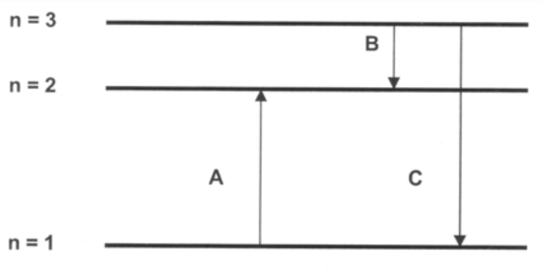
$c = f\lambda$, so $f = c/\lambda = (3 \times 10^8 \text{ m/s})/(6.56 \times 10^{-7} \text{ m}) = 4.57 \times 10^{14}$ Hz.

Now, the energy of this photon can be found from the equation $E = hf$:

$$E = hf = (4.14 \times 10^{-15} \text{ eV-s}) \times (4.57 \times 10^{8} \text{ Hz}) = 1.89 \text{ eV}.$$

This must be equal to the change in energy level of the electron that emits this wave. By looking over the energy level diagram, we can see that $\Delta E_{23} = 1.89$ eV; this tells us that it must be an electron dropping down from the $n = 3$ level to the $n = 2$ level emits an e-m wave with a wavelength of 656.3 nm.

Here is a schematic energy level diagram of some unknown atom:



As in the diagram of the hydrogen atom, the spacing between the lines is proportional to the differences in energy levels represented by the lines. So, the spacing between $n = 1$ and $n = 2$ is larger than that between $n = 2$ and $n = 3$, which means that $\Delta E_{12} > \Delta E_{23}$. Transition "A" shows an electron increasing its energy level; in order to do this, it must have absorbed some energy – either from an incoming e-m wave, or from being struck by a moving electron. If it absorbs an e-m photon, the incoming photon must have an energy exactly equal to ΔE_{12} . If it does not, the transition cannot take place. Similarly, transition "B" represents the emission of an e-m wave.

Question: Is the frequency of the e-m wave emitted in transition B greater than, less, than, or equal to the frequency of the e-m wave absorbed in transition A?
Answer: The frequency of the "B" photon is less than that of the "A" photon, because its energy is less.

Question: How does the wavelength of the photon emitted in transition C compare to that emitted in transition B?
Answer: The wavelength of the C photon is smaller than that of the B photon; that is because the frequency (and the energy) of the C photon is greater, and a greater frequency corresponds to a smaller wavelength.

Questions

1. Which of the following would have the highest-energy photons:

(A) radio waves (B) infrared waves (C) visible light waves
(D) X-rays (E) gamma rays

2a. What is the energy in joules of the photons associated with red light (wavelength $\lambda = 700$ nm)?

(1) 1.6×10^{-19} J (2) 2.8×10^{-19} J (3) 5.0×10^{-19} J

2b. What is the energy in electron-volts of the photons associated with red light (wavelength $\lambda = 700$ nm)?

(1) 1.77 eV (2) 3.1 eV (3) 4.2 eV (4) 6.4 eV

3a. What is the energy in joules of the photons associated with blue light (wavelength $\lambda = 400$ nm)?

(1) 1.6×10^{-19} J (2) 2.8×10^{-19} J (3) 5.0×10^{-19} J

3b. What is the energy in electron-volts of the photons associated with blue light (wavelength $\lambda = 400$ nm)?

(1) 1.77 eV (2) 3.1 eV (3) 4.2 eV (4) 6.4 eV

4. A 100-watt incandescent light bulb puts out about 5% of its power as visible light. Using an average wavelength of visible light, about how many photons is it emitting every second?

(1) 10^3 (2) 10^7 (3) 10^{11} (4) 10^{15} (5) 10^{19}

5. (a) The earth is 150 million kilometers away from the sun and receives about 1370 watts of solar energy over each square meter (at the top of the atmosphere) of surface perpendicular to the sun's rays. Assuming the sun is radiating isotropically (the same in all directions), what is the total number of watts radiated by the sun? Hint: the surface area of a sphere of radius R is $4\pi R^2$.

(1) 4×10^{26} W (2) 5×10^{28} W (3) 6×10^{30} W (4) 7×10^{32} W

(b) Using the average wavelength and frequency of visible light, about how many photons is the sun emitting each second?

(1) 10^{25} (2) 10^{35} (3) 10^{45} (4) 10^{55} (5) 10^{65}

6. Suppose two energy levels in an atom have energies of -5.67 eV and -12.88 eV. What will be the energy (in electron-volts) of a photon emitted when the atom changes from the higher energy state to the lower energy state?

What will be the frequency of this photon?

What will be the wavelength associated with this photon?

7. On the energy diagram for the hydrogen atom, shown a few pages back, there are six energy levels denoted $n = 1$ through $n = 6$. Suppose there are electrons making transitions from all of these levels to all possible levels of lower energy. How many different photons could result? Hint: Figure out how many different energies could result from the transitions.

(1) 5 (2) 10 (3) 15 (4) 20 (5) 25 (6) 30

Which transition (give the initial and final values of n) would have the lowest frequency photon?

From $n_i =$ to $n_f =$

Which transition would have the second highest frequency photon?

From $n_i =$ to $n_f =$

Which transition would have the highest frequency photon?

From $n_i =$ to $n_f =$

NUCLEAR STRUCTURE

In the previous section on photons and atomic spectra, we discussed the electrons in the atom, but we did not say anything regarding the atomic nucleus. There are no electrons in the nucleus and, by comparison to the amount of space occupied by the electrons, the nucleus is extremely tiny. Nonetheless there are powerful forces at work in the nucleus, and as a result many important phenomena have their origins there.

Structure of the Nucleus: The nucleus [plural: nuclei], which is the "core" of the atom, is composed of particles called "***nucleons***," of which there are two types:

(1) ***protons*** (positively charged); the number of protons in a nucleus is called the "***Atomic Number***" [symbol: Z], and identifies which element corresponds to that particular nucleus.

(2) ***neutrons*** (no electric charge); the number of neutrons in a nucleus is called the "neutron number."

Different ***isotopes*** of a single element all have the same Z, but different numbers of neutrons.

The nucleons are held together by the ***"strong" force***, which is also called the "nuclear" force. This is the strongest force known in nature – about 100 times stronger than the electromagnetic force. The reason that we are not normally aware of its presence is that it has a very short range: its effects are only significant over distances smaller than about 10^{-15} m. The nucleus itself is approximately 10^{-14} m in diameter (larger for nuclei with more protons), while the diameter of an atom is much larger: approximately 10^{-10} m.

It is the powerful nuclear force that keeps the protons in the nucleus bound tightly together; without it, repulsive electrical forces would push the protons far apart. Instead of the different parts of the nucleus breaking apart on their own, a great deal of external energy is required to disassemble a nucleus. The source of nuclear energy is the strong nuclear force; this energy can be released in processes such as "***fission***" (the splitting of a nucleus into two parts) and "***fusion***" (the joining of two nuclei to form one larger one).

The "ordinary" state of an atom is to be electrically neutral; in that case, the number of electrons must equal the number of protons. (Atoms may also exist as ions; in that case, there are more or fewer electrons than there are protons.) It is the electrons – their number and their arrangement in the atom - that determine the chemical properties of the atom. For that reason, the atomic number Z – which is equal to the number of protons – determines the chemical properties of the neutral atom. Thus, different elements – each of which has a different value of Z – have different and distinct chemical properties.

Radioactive Decay: Certain types of nuclei are said to undergo radioactive "decay" when they transform into another type of nucleus. (Nuclei that decay are said to be "unstable.") The transformation, or decay, is accompanied by emission of certain types of particles ("alpha" and "beta" particles), or of a gamma ray. An ***alpha particle*** is composed of two neutrons and two protons bound together; it is identical to the nucleus of a helium atom. A ***beta particle*** is actually just an electron; this electron is produced by certain types of nuclear interactions, and is not a permanent component of the nucleus. The gamma rays that are emitted during radioactive decay are high-energy electromagnetic photons; they are produced when the nucleons drop from one energy level to a lower one. (Just as the electrons in an atom have certain specific energy levels, so too do the nucleons.)

Alpha decay can be expressed by this reaction: $\mathbf{{}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha}$**.**

This expresses the fact that the nucleus of element X, of atomic number Z and nucleon number A has emitted an alpha particle (a helium nucleus with $Z = 2$ and $A = 4$), leaving a nucleus of element Y with atomic number $Z - 2$ and nucleon number $A - 4$. Note that the atomic number on then left equals the sum of those on the right, and the same for the nucleon number.

Example: What is the reaction by which radium-226 emits an alpha-particle?

Solution: Radium is element number 88, so, recognizing that element number 86 is radon (symbol: Rn), the reaction is:

$$^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\alpha$$

Ordinary beta decay can be expressed by this reaction: $\mathbf{{}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta}$**.**

This expresses the fact that the nucleus of element X, of atomic number Z and nucleon number A has emitted an beta particle or electron, which can be considered to have $Z = -1$ and $A = 0$), leaving a nucleus of element Y with atomic number $Z + 1$ and nucleon number A. Effectively, a neutron in the nucleus has changed into a proton, with the emission of an electron. Note that again the atomic number on then left equals the sum of those on the right, and the same for the nucleon number.

There is another form of beta decay in which a positron (written β^{+}) is emitted as a proton is converted into a neutron. This decay can be expressed by the reaction

$$\mathbf{{}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta^{+}}$$

Example: Write down the reactions by which the radium-228 nucleus undergoes beta decay and the mercury-185 nucleus undergoes positron emission.

Solution: The beta decay of radium-228 produces an actinium nucleus:

$$^{228}_{88}\mathrm{Ra} \rightarrow {}^{228}_{89}\mathrm{Ac} + {}^{0}_{-1}\beta$$

and the positron decay of mercury-185 produces a nucleus of francium:

$$^{228}_{88}\mathrm{Ra} \rightarrow {}^{228}_{87}\mathrm{Fr} + {}^{0}_{+1}\beta^{+}$$

Radioactivity – the decay of unstable nuclei and the accompanying emission of particles and gamma rays – has many important practical uses. Radioactive materials are used as "tracers" for medical diagnosis, and they are also used for therapeutic purposes (such as the destruction of malignant tumors). Radioactivity can also be used to find the ages of both biological materials and geological samples. With these methods, ages up to several billion years can be reliably determined.

It is not possible to determine when one particular unstable nucleus will decay. However, it is possible to determine to a very high degree of accuracy what proportion of a very large sample of unstable nuclei of any given type will decay each second. For instance, if we have 1000 nuclei of type "A," we might find that 4% of them will decay each second. (We will assume here that they each decay into a single nonradioactive nucleus.) In this example, we would find that of the original sample of 1000 nuclei, 40 of them will decay in the first second. For any other sample consisting of 1000 nuclei of type "A", we can be confident that very close to 40 of them will decay in one second. We would say that the "***decay constant***" of this particular nucleus is 4%/s [four percent per second]. Using the symbol λ for decay constant, we would say that $\lambda = 0.04$/s for this nucleus. [We can also write this as $\lambda = 0.04\ \mathrm{s}^{-1}$].

Some symbols we will use:

$\boldsymbol{N}$: number of radioactive nuclei of a given type present at a particular moment in time. N is a function of time: $N = N(t)$.

$\boldsymbol{\lambda}$ ["decay constant"]: fraction of nuclei present which decay per unit time. Note that in this context, λ does not represent a wavelength! If we express λ in units of s^{-1}, we have this important relationship:

$\boldsymbol{n}$: number of radioactive decays [and so we have: number of decays per unit time $= n/\Delta t$]

$$\boxed{\textbf{decays per second} = \boldsymbol{\lambda N}}$$

$\boldsymbol{\Delta N}$: change in value of N. [A decrease in N would correspond to a negative value of ΔN.] If there are "n" decays in time Δt, the value of N will decrease by n. Therefore, we can see that:

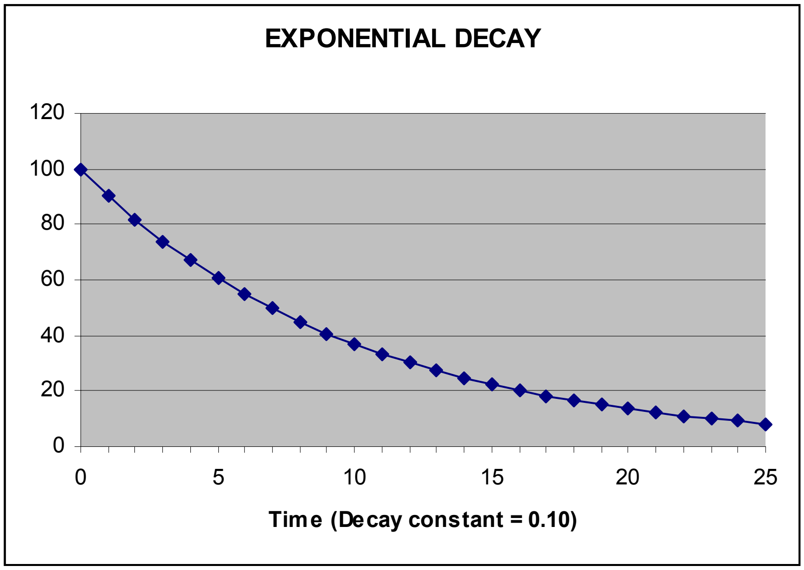
$$\frac{\Delta N}{\Delta t} = \frac{-n}{\Delta t} = -\lambda N$$

This equation leads to the following mathematical expression for $N(t)$:

$$N(t) = N_0 e^{-\lambda t}$$

If a certain number of radioactive nuclei are collected together, some of them will decay each second. Ordinarily, they decay into a single nonradioactive nucleus. (This is the situation we will consider here.) Therefore, as time goes on, the number of radioactive nuclei remaining declines. What stays the same, however, is the decay constant λ; that is a property of that type of nucleus, and is unrelated to the number of such nuclei that are present.

Because the decay constant λ does not change, the ***fraction*** of the radioactive nuclei that decay each second does not change. But that is not the fraction of the original number of nuclei – it is the fraction of the number present at any given moment. That is, the number of decays per second is equal to λN. So, if $\lambda = 0.03$/s, 3% of the radioactive nuclei present – call this number "N_1" - will decay in one second. But then, after that one second, there are fewer radioactive nuclei present; call that new number "N_2"; 3% of N_2 will decay in the next second. (The total number of nuclei – radioactive plus nonradioactive – remains constant, according to our assumptions here.) For this reason, the number of decays per second declines in direct proportion to the number of radioactive nuclei present. Because the number that decay each second gets smaller as time goes on, the rate of decrease of radioactive nuclei itself decreases. This leads to a graph of $N(t)$ that decreases as a function of time – but not in a straight line. Rather, the graph is curved, and looks something like this:



The time required for a given sample of radioactive nuclei to decrease in number by 50% is a characteristic property of each type of nucleus. It is related to the decay constant, and so does not vary when the number of nuclei present changes. This time is called the "***half-life***" [symbol: $t_{1/2}$]; it is inversely related to the decay constant. (A larger decay constant corresponds to a smaller half-life.)

Radioactive Dating: Radioactive materials are often used to provide estimates of the ages of substances that contain those radioactive materials. A particularly important example is radiocarbon dating, which makes use of the radioactive nucleus ^{14}C that has a half-life of 5730 years. (The number "14" refers to the total number of nucleons – protons [6] plus neutrons [8] – in this nucleus. The common, nonradioactive form of carbon is ^{12}C, the one that constitutes practically all the carbon nuclei in our bodies; it has 6 protons and 6 neutrons.)

The relative proportions of ^{14}C and ^{12}C in the atmosphere have been fairly stable for the past 50,000 years or so. In a sample of 10^{12} nuclei, approximately one will be ^{14}C, and the rest will be ^{12}C. Plants absorb carbon (in the form of carbon dioxide) from the air and animals eat plants. As long as the organisms are alive, the ratio of ^{14}C to ^{12}C in their tissues remains at the atmospheric ratio of 1 in 10^{12}. However, when the organism dies, the radioactive carbon is no longer replenished from the atmosphere and so its proportion decreases. By measuring the relative proportions of the two types of carbon in a sample of biological material, the age of the sample can be determined with great accuracy (up to approximately 50,000 years).

For instance, suppose a sample of material is examined in which only one ^{14}C nucleus is present for every 4×10^{12} ^{12}C nuclei. This is only ¼ of the usual ratio. It must be that the rest of the ^{14}C nuclei which were originally present have decayed away. Now, it takes 5730 years − one half-life − for the number of ^{14}C nuclei to decrease by half. That is, we started out with a concentration of ^{14}C nuclei that was equal to one in 10^{12}; after 5730 years have elapsed, that concentration will have declined to only one-half in 10^{12}. Of course you can't have a half a nucleus; this just means that the concentration has been reduced to one for every 2×10^{12} ^{12}C nuclei. If we observe a ratio of one for every 4×10^{12} ^{12}C nuclei, that must mean that two half-lives have elapsed − 11,460 years.

Question: Suppose a fresh sample of biological material is examined that shows 200 decays per second of ^{14}C nuclei. (This is either living tissue, or something that died within the past few years.) How many decays per second would be observed if this same sample were examined 5730 years later?
Answer: 100 decays per second. The number of radioactive nuclei has decreased; there are now only half as many as there were originally. The decay constant λ is still the same, so the fraction of the number present that decay each second is the same as before. But since the number present has decreased to half of its original value, the number of decays per second must also have decreased to 50% of its original value.

Radioactive Decay: Certain "unstable" nuclei are said to undergo radioactive "decay" when they transform into another type of nucleus, accompanied by emission of certain types of particles ("alpha" and "beta" particles), or of a gamma ray. An alpha particle is composed of two neutrons and two protons bound together; it is identical to the nucleus of a helium atom. A beta particle is actually just an electron.

The decay constant λ is the fraction of nuclei that decay per second, so the number of decays per second is λN if there are N radioactive nuclei.

The half-life is the time it takes for half the nuclei to decay: $t_{\frac{1}{2}} = 0.693/\lambda$, where 0.693 is the natural logarithm of 2. After two half-lives (time $2t_{\frac{1}{2}}$) only 1/4 of the nuclei remain, after three half-lives 1/8, etc.

The number of nuclei decreases exponentially. The number of nuclei left at time t, if N_0 is the number of nuclei at time $t = 0.00$ s, is given by $N(t) = N_0 e^{-0.693t/t_{\frac{1}{2}}}$.

Example 1. A radioactive nucleus has a decay constant of 0.00056/s. Approximately how much of a sample of 15.00 grams of the radioactive element will remain undecayed one hour later?

Solution: Instead of the number of nuclei we can use its mass, since the two are proportional to one another.

The mass that has not decayed one hour $= 1$ h $\times$ 60 min/h $\times$ 60 s/min $= 3600$ s later is then

$$M = M_0 e^{-\lambda t} = (15.00\text{ g})e^{-(0.00056/\text{s})(3600\text{ s})} = (15.00\text{ g})e^{-2.016} = 0.133\text{ g}.$$

Example 2. A radioactive nucleus has a half-life of 10.00 hours. What fraction of a sample of this nucleus will remain after 1 hour?

Solution: The fraction remaining is

$N(t)/N_0 = e^{-0.693t/t_{\frac{1}{2}}} = e^{-0.693\,(1\text{ h})/(10\text{ h})} = e^{-0.0693} = 0.933$, or 93.3%.

Example 3. A radioactive nucleus has a half-life of 10.00 hours. What fraction of a sample of this nucleus will decay during 100 hours?

Solution: The fraction decaying is $1 -$ (fraction remaining):

$1 - N(t)/N_0 = 1 - e^{-0.693t/t_{\frac{1}{2}}} = 1 - e^{-0.693\,(100\text{ h})/(10\text{ h})} = 1 - e^{-6.93} = 1 - 0.000978$

$= 0.999022$ or 99.90%.

Another approach: 100 hours is 10 half-lives, so the fraction remaining should be $(1/2)^{10} = 0.0009765625$ and the fraction decaying is $1 - 0.0009765625 = 0.999023$ or 99.90%.

QUESTIONS

1. Consider the isotope denoted by $^{225}_{89}$Ac.

It has __________ electrons, _____________ protons, and ___________ neutrons.

2. This atom is radioactive and decays by alpha decay with a half-life of 10.0 days. Write down the decay of this atom and determine the new atom that results.

3. The new atom has ____________ protons and _____________ neutrons in its nucleus.

4. What is the decay constant of the original actinium isotope?

5. How much of an initial amount of this isotope will be left (undecayed) after two weeks?

6. How long will it take for 1/3 of an initial amount of this isotope to decay?