

Topic 8. Wave Nature of Light

Topics 8a. Light interference through slits

Electromagnetic waves, like sound waves, can exhibit interference effects. When two or more waves pass through the same point in space, the electric and magnetic fields at that point are the vector sums of the individual electric and magnetic fields due to the different waves:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots \text{ and } \vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \dots$$

Just as sound waves can exhibit constructive and destructive interference, so can electromagnetic waves. Interesting interference effects occur when two light waves have exactly the same frequency (and thus wavelength) and arrive from the same source by two different paths. Examples of interference are:

- (1) Two-slit interference, when the light passes through two openings and then is viewed on the other side.
- (2) Single-slit diffraction, when the light passes through different parts of the same slit and then is viewed on the other side.
- (3) Thin-film interference, when the light reflects off both the top and bottom surfaces of a thin film of a different material than the material from which the light is incident.
- (4) Interference when the light has been split and reflected off different mirrors before coming together again, as in a Michelson interferometer.

CONDITIONS FOR INTERFERENCE

The conditions at a point P for completely constructive and completely destructive interference of waves of wavelength λ from a source by two different paths of path length difference δ are very simple, in principle:

For completely constructive interference: $\delta = m\lambda$, where $m = 0, \pm 1, \pm 2, \dots$

For completely destructive interference: $\delta = (m + \frac{1}{2})\lambda$, where $m = 0, \pm 1, \dots$

The light travels from the source to the point P by two different paths; δ is the difference in lengths of the two paths. For example, if light from a source goes straight to a screen over a path 1.000 m long, and other light from the source goes in a different direction, strikes a mirror, and comes back to that same point on the screen, with a total path length of 1.234 m, then the path length difference is $\delta = 1.234 \text{ m} - 1.000 \text{ m} = 0.234 \text{ m}$.

Waves (such as light or water waves) passing through openings in a barrier or around a barrier will spread out to either side on the far side of the opening. This is not very noticeable when the sizes or separation of the openings are much greater than the wavelength, but is very noticeable when they are small. This makes interference effects possible. The light does not travel just in straight lines.

Points where completely constructive interference occur are bright, while those where completely destructive interference occurs are dark. The resulting patterns are often described as "bright and dark" fringes.

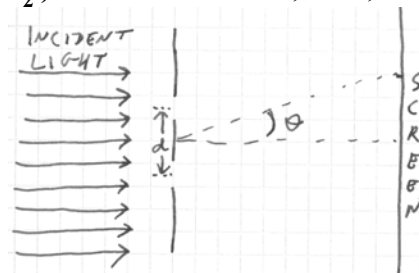
The pattern of red laser light passing through a pair of narrow slits. Instead of just a pair of bright lines corresponding to light having passed directly through each slit, one finds this complex pattern of bright red fringes and dark fringes in between.

YOUNG'S DOUBLE-SLIT EXPERIMENT

In Young's double-slit experiment, a fringe pattern is produced by light which passes through two openings (slits) separated by a distance d . Fringes are regions in which the light is at its brightest or darkest. Straight across from the middle of the slits one finds a bright fringe due to constructive interference. For light rays whose paths deviate from the straight-forward direction by an angle θ , the extra path length from the farther slit is $d \sin \theta$, so the angles θ_m at which bright or dark fringes occur are:

- Maxima (bright fringes) if $d \sin \theta_m = m\lambda$
- Minima (dark fringes) if $d \sin \theta_m = (m + \frac{1}{2})\lambda$ where $m = 0, \pm 1, \pm 2, \dots$

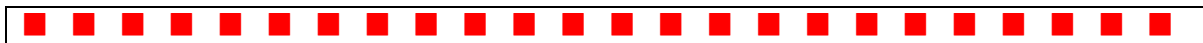
Young's experiment is set up so that light from a distant source reaches two narrow parallel slits in phase. However, a phase difference occurs on the screen because of the path difference to the screen from the two slits (except right at the center). If the screen is far away the path difference δ is approximately $d \sin \theta$.



In Young's double-slit experiment, a fringe pattern is produced by light passed through two openings (slits) separated by a distance d . For light deviation from the straight-forward direction by an angle θ :

- Maxima (bright fringes) occur if $d \sin \theta = m\lambda = \delta \approx d \sin \theta$
- Minima (dark fringes) occur if $(m + \frac{1}{2})\lambda = \delta \approx d \sin \theta$ where $m = 0, \pm 1, \pm 2, \dots$

The appearance of the pattern on the screen is a series of equally-spaced bright fringes, looking more or less as shown below. Note that it is essentially impossible to determine which bright fringe is the central fringe.



Note that if the separation of the slits is reduced, the bright fringes will occur at larger angles, meaning that the pattern widens, which is counterintuitive:



QUESTIONS

Consider two slits which have a separation (between their centers) of $40.0 \mu\text{m}$. Light of wavelength $\lambda = 500 \text{ nm}$ passes through the slits and lands on a screen 2.00 m away.

(1) On the screen, directly across from the center of the region between the two slits, one finds:

- (1) No light (a dark fringe) (2) A bright fringe
(3) Impossible to predict

(2) The smallest angle (on either side of the center) at which an interference maximum occurs is:

- (1) 0.36° (2) 0.72° (3) 1.08° (4) 1.43° (5) 2.15°

(3) The distance on the screen of the first interference maximum, measured from the center of the pattern, is:

- (1) 2.5 cm (2) 5.0 cm (3) 7.5 cm (4) 10 cm (5) 12.5 cm

(4) The smallest angle (on either side of the center) at which an interference minimum occurs is:

- (1) 0.36° (2) 0.72° (3) 1.08° (4) 1.43° (5) 2.15°

(5) Suppose the screen has a finite width, and is located at a distance such that it subtends angles, as measured from the center of the slits, from -12.0° to $+12.0^\circ$. How many bright interference fringes would be found on the screen?

- (1) 16 (2) 17 (3) 32 (4) 33 (5) 34

(6) Suppose that light of wavelength $\lambda = 400 \text{ nm}$ were passed through the same slits.

- (1) The interference maxima would occur at the same angles as before.
(2) The interference maxima would occur at smaller angles than before.
(3) The interference maxima would occur at larger angles than before.
(4) The interference maxima would occur at different angles, but it is not possible to determine them.

(7) Suppose that light of wavelength $\lambda = 600 \text{ nm}$ were passed through the same slits.

- (1) The interference maxima would occur at the same angles as before.
(2) The interference maxima would occur at smaller angles than before.
(3) The interference maxima would occur at larger angles than before.
(4) The interference maxima would occur at different angles, but it is not possible to determine them.

MULTIPLE SLITS

When coherent monochromatic light of wavelength λ shines on a system of N evenly-spaced slits separated by distances d , the principal maxima occur at the same angles θ (given by $d \sin \theta = m\lambda$) as in the case $N = 2$ (Young's double-slit experiment), but the maxima become progressively sharper and sharper, with more and more small secondary maxima in between the principal maxima.

The interference pattern becomes sharper (with smaller but brighter bright fringes) when the number of slits increases. It resembles more:



Consider N slits uniformly separated by a distances d . Principal maxima occur at the same angles as for two slits:

$$\phi = 2m\pi \quad (m = 0, 1, 2, 3, \dots) \quad \text{and} \quad d \sin \theta = m\lambda \quad (m = 0, 1, 2, 3, \dots)$$

Here ϕ is the phase difference between two adjacent light beams.

There are also minima for $\phi = 2p\pi/N$ where $p = 1, 2, 3, 4, \dots$ but $p \neq N, 2N, 3N, \dots$ There are $N - 1$ of these minima between each pair of principal maxima, so there must also be $N - 2$ secondary maxima between each pair of principal maxima. Consequently, the interference maxima for the multiple-slit case occur at the same positions as those for the 2-slit case, but they become sharper and sharper as the number of slits increases.

GRATINGS

A grating consists of thousands of lines or grooves on a glass plate or on plastic.

The interference maxima for a grating with line spacing d occur at angles θ given by $d \sin \theta = m\lambda$, where m ($= \pm 1, \pm 2, \pm 3, \dots$) is called the "order" of the spectrum.

A grating is effectively a multiple-slit system with a very large value of N . The interference maxima occur at exactly the same angles as if there were but two lines a distance d apart. However, these maxima are much sharper in a grating, because the extra lines lead to more secondary maxima.

FRAUNHOFER AND FRESNEL DIFFRACTION

Interference also occurs between light waves passing through different parts of the same opening onto a screen. Constructive interference occurs straight across from the opening, but not to the sides.

Fresnel diffraction is diffraction through an opening when the source on one side and the screen on the other side are at finite distances from the opening. This is a complicated situation to analyze, and we won't discuss it in this course.

Fraunhofer diffraction is diffraction through an opening when the source on one side and the screen on the other side are at infinite distances from the opening, or, equivalently, the light rays are parallel to the axis. This is the case we will discuss.

SINGLE-SLIT DIFFRACTION

The diffraction pattern of a single narrow rectangular slit of width a , for light of wavelength λ , has a central maximum at $\theta = 0$ and minima at values of θ for which

$$a \sin \theta = m\lambda \quad (m = \pm 1, \pm 2, \pm 3, \dots)$$

Secondary maxima occur in between the minima, approximately, but not exactly, at angles θ for which $a \sin \theta = m\lambda$ and $m \approx \pm 1.5, \pm 3.5, \dots$

The minima of a single-slit diffraction pattern, for a long narrow slit of width a , occur when:

(1) the path difference between waves from one end of the slit are out of phase with those from the middle of the slit, because then one half of the slit destructively interferes with the other half of the slit. The condition for this is that

$$\frac{1}{2}a \sin \theta = \frac{\lambda}{2} \quad \text{or} \quad a \sin \theta = \lambda$$

or (2) the width of the slit can be divided into $m = 2, 3, 4, 5, \dots$ equal parts for which the waves in each part interfere destructively as in part (1). This means

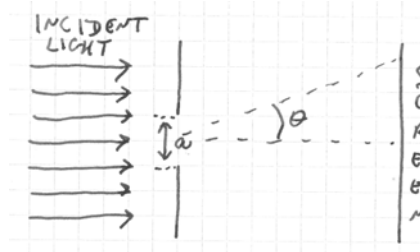
$$\frac{1}{2} \frac{a}{m} \sin \theta = \frac{\lambda}{2} \quad \text{or} \quad a \sin \theta = m\lambda$$

In general, then the diffraction minima are at

$$a \sin \theta = m\lambda \quad (m = \pm 1, \pm 2, \pm 3, \dots)$$

There is a maximum (the central maximum) at $\theta = 0$, and secondary maxima approximately half-way between the diffraction minima.

Note that in double-slit interference and single-slit diffraction the angle θ corresponds to a maximum, called the central maximum. Elsewhere, there is a simple condition for the interference *maxima* and for the diffraction *minima*.



DIFFRACTION AND INTERFERENCE

The real pattern from a pair of slits exhibits both diffraction (due to the width a of the slits) and interference (due to the separation d of the slits). Since $d > a$, the diffraction pattern is "wider" than the interference pattern, so the effect is that there is a diffraction pattern divided up into narrower bright and dark interference fringes. This is shown in the photograph below, where $d = 6a$. Since every sixth interference maximum (at $\sin \theta = m\lambda/d = m\lambda/6a$ for $m = 6, 12, 18, \dots$) coincides with a diffraction minimum ($\sin \theta = m'\lambda/a$ for $m' = 1, 2, 3, \dots$), it is actually suppressed and appears to be "missing."

Diffraction-interference pattern produced by a pair of slits with separation-to-width (d/a) ratio of 6. Note that every sixth interference maximum appears to be missing because it coincides with a diffraction minimum.

QUESTIONS

Consider a narrow slit of width of $10.0 \mu\text{m}$. Light of wavelength $\lambda = 500 \text{ nm}$ passes through the slit and lands on a screen 2.00 m away.

(1) On the screen, directly across from the center of the slit, one finds:

- (1) No light (a dark fringe) (2) A bright fringe
(3) Impossible to predict

(2) The smallest angle (on either side of the center) at which a diffraction minimum occurs is:

- (1) 0.36° (2) 0.72° (3) 1.08° (4) 1.45° (5) 2.9°

(3) The distance on the screen of the first diffraction minimum, measured from the center of the pattern, is:

- (1) 10 cm (2) 20 cm (3) 30 cm (4) 40 cm (5) 50 cm

(4) The smallest angle (on either side of the center) at which an secondary diffraction maximum occurs is approximately:

- (1) 1.5° (2) 2.9° (3) 4.3° (4) 5.7° (5) 7.1°

(5) Suppose the screen has a finite width, and is located at a distance such that it subtends angles, as measured from the center of the slits, from -12.0° to $+12.0^\circ$. How many complete bright diffraction fringes would be found on the screen?

- (1) 5 (2) 6 (3) 7 (4) 8 (5) 9

(6) Suppose that light of wavelength $\lambda = 400 \text{ nm}$ were passed through the same slits.

- (1) The diffraction minima maxima would occur at the same angles as before.
(2) The interference maxima would occur at smaller angles than before.
(3) The interference maxima would occur at larger angles than before.
(4) The interference maxima would occur at unknown but different angles.

(7) Suppose that light of wavelength $\lambda = 600 \text{ nm}$ were passed through the same slits. (Same choices as in #6)

CIRCULAR APERTURES

The diffraction pattern of light of wavelength λ for a circular aperture of diameter a has a central maximum and a first diffraction minima in the form of a circle at the angle θ given by $\sin \theta = 1.22 \lambda/a$.



The photograph above shows the diffraction pattern for a circular aperture. The central maximum is a circle, and the diffraction minima and secondary maxima are rings of bright and dark regions. You can see three maxima (the central and two secondary ones) and three minima in this photograph.

The analysis that yields this result is complicated, and involves Bessel functions, which are beyond the scope of this course. Nevertheless, this is a useful result.

THE RAYLEIGH CRITERION

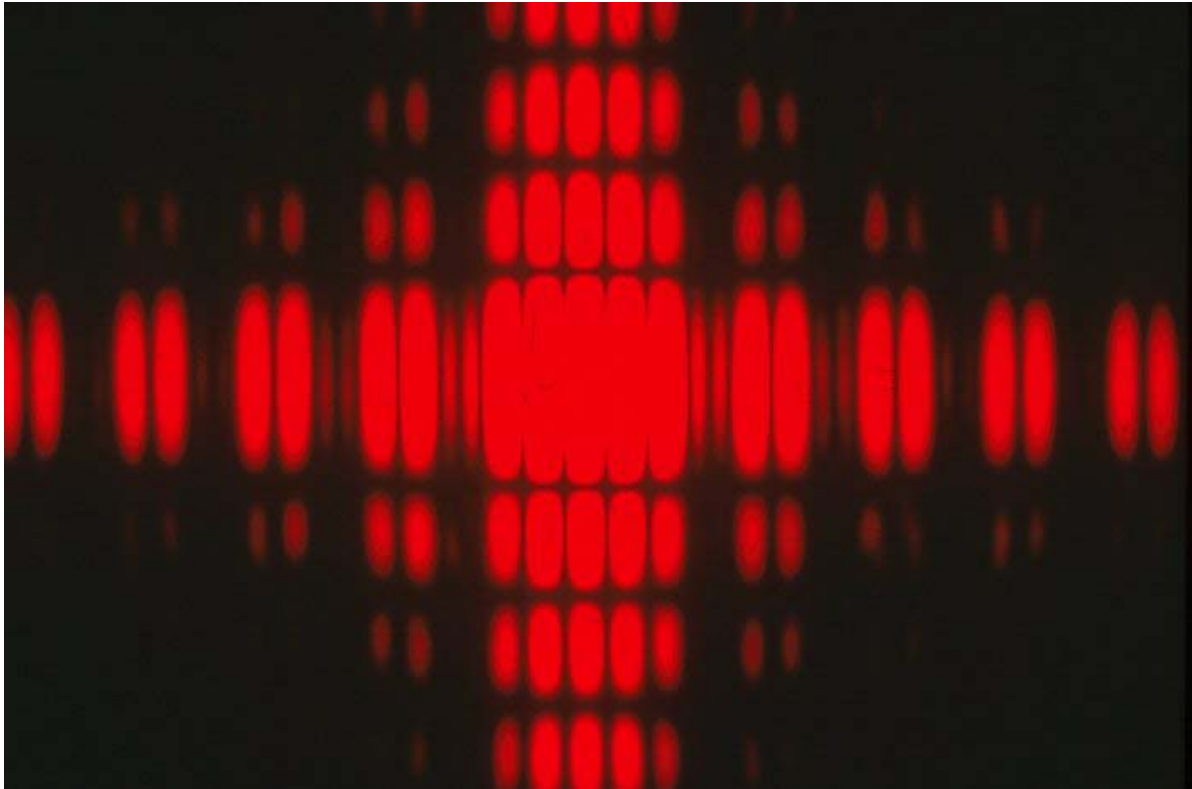
Rayleigh's criterion, which works well in practice, is that two point sources can just barely be distinguished as two sources if the central diffraction maximum of one just coincides with the first diffraction minimum of the other.

Viewed through a circular lens or other aperture of radius a , two sources can thus just barely be resolved if their angular separation is

$$\theta = \sin^{-1}(1.22 \lambda/a) \approx 1.22 \lambda/a.$$

The resolution of the light from two sources is impeded by diffraction. When two sources have a large angular separation, the diffraction patterns are far apart, but when they have a small angular separation, the diffraction patterns overlap. An example is a car approaching you at night on a long road. When the car is far away, its headlights appear to be just a single bright light. As the car gets nearer, so that the angular separation of the lights is greater, the headlights look more like a dumbbell, and eventually they look like two separate bright lights.

Under what condition can the images be resolved? Rayleigh proposed the following criterion: that two point sources can barely be distinguished if the central diffraction maximum of one just coincides with the first diffraction minimum of the other. This is about right. Viewed through a circular lens or other aperture of radius a , such as the human eye or a telescope, two sources can just be resolved if their angular separation $\theta = \sin^{-1}(1.22\lambda/a) \approx 1.22\lambda/a$. Since this criterion is approximate, we might just as well use $\theta \approx \lambda/a$.



This extraordinary photograph shows the diffraction-interference pattern for red laser light passing through two square apertures (a by a) whose centers are separated by $d = 3a$.

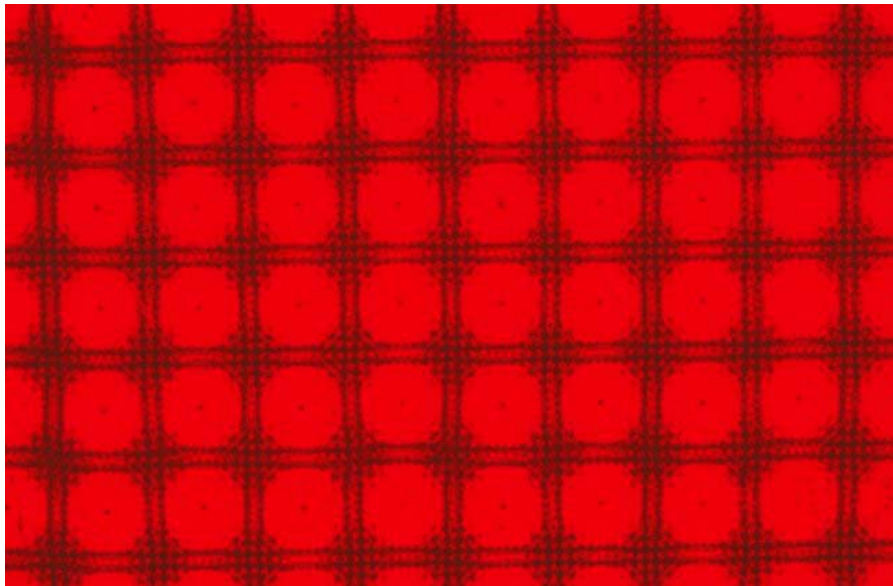
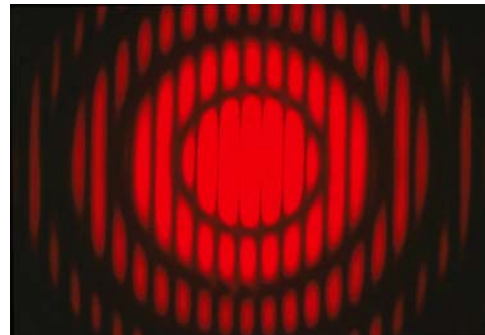
Note that the pattern is two dimensional, because the apertures are not narrow slits, but squares. The main diffraction pattern due to the square shape of the apertures is visible along the left-to-right and top-to-bottom directions in the plane of the screen. Superimposed upon it is the interference pattern due to the fact that there are two apertures.

Since $d = 3a$, every third interference fringe is missing, as is most easily seen just above and below the center (brightest) line of the pattern. Along the center line the missing interference pattern appears to be replaced by two narrower interference fringes, which are just the outer (not completely suppressed) parts of the missing fringe; these are visible because the photograph was overexposed to show them.



The photograph to the left shows the "shadow" of a needle. Interference of light waves coming from the two sides of the needle produces a bright center line down the middle of the shadow, causing the needle to appear "split."

Look at the pattern shown on the right and guess what produced it. The three-word answer (to be revealed in class) is:



What is the world caused this complex pattern??? Note the weird dark spot at the center of each of the bright regions.

14b. Thin-Film Interference

Interference by thin films occurs when light incident on a thin transparent film (such as oil floating on water) is reflected from both the top surface of the thin film and the bottom surface (after refraction into the film), and the two reflected beams interfere constructively or destructively when viewed from above the film. The most common example of this phenomena is the colored nature of the thin film, the colors corresponding to the wavelengths for which constructive interference result.

air ($n = 1.00$)

oil ($n = 1.40$)

$\updownarrow t = \text{thickness of oil film}$

water ($n = 1.33$)

Destructive or constructive interference in thin films can be determined using these facts:

(1) Light undergoes a phase change of π radians (180°) when it is reflected at an interface off a medium of higher index of refraction; otherwise, there is no phase change.

(2) The wavelength λ in a medium of refractive index n is related to the wavelength λ_{air} in air by $\lambda = \lambda_{\text{air}}/n$.

Example 1: Suppose the phase changes upon reflection are different at the top and bottom surfaces of the thin film of thickness t : one is 0° and the other is 180° . Then for light that is normally incident (the only case we'll consider in this course),

Constructive interference occurs if $2t = (m + \frac{1}{2})\frac{\lambda_{\text{air}}}{n}$ for $m = 0, 1, 2, \dots$

(because this introduces another phase change of 180° to bring the reflected waves into phase)

and Destructive interference occurs if $2t = m\frac{\lambda_{\text{air}}}{n}$ for $m = 0, 1, 2, \dots$

(because this does not change the phase, leaving a total change of 180°)

Example 2: Suppose the phase changes upon reflection are the same at the top and bottom surfaces of the thin film of thickness t , normally meaning both are 180° . Then for light that is normally incident,

Destructive interference occurs if $2t = (m + \frac{1}{2})\frac{\lambda_{\text{air}}}{n}$ for $m = 0, 1, 2, \dots$

(because this introduces another phase change of 180° to bring the reflected waves out of phase)

Constructive interference occurs if $2t = m\frac{\lambda_{\text{air}}}{n}$ for $m = 0, 1, 2, \dots$

(because this does not change the phase, leaving no phase change)

QUESTIONS

1. For each of the following, determine the phase change of the reflected light.

Answers: (1) 0° (2) 90° (3) 180°

- (a) Light incident from air ($n = 1.00$) onto glass ($n = 1.50$)
 - (b) Light incident from glass onto air
 - (c) Light incident from air onto water ($n = 1.33$)
 - (d) Light incident from water onto air
 - (e) Light incident from water onto glass
 - (f) Light incident from glass onto water
-

2. Suppose several immiscible liquids of different densities float on top of one another. From the top down, beginning with air ($n = 1.00$), the liquids have indices of refraction of 1.21, 1.46, 1.18, and 1.33. At which surfaces would the reflected light be changed in phase, and by how much in each case?

3. Light is normally incident from air ($n = 1.00$) onto a thin film of oil ($n = 1.40$) floating on water.

(a) What is the phase change of the ray reflected back into the air from the top of the oil film?

- (1) 0° (2) 180° (3) An angle between 0° and 180°

(b) What is the phase change of the ray reflected back into the oil from the bottom of the oil film?

- (1) 0° (2) 180° (3) An angle between 0° and 180°

(c) For what thickness of film will light of wavelength 560 nm (in air) exhibit constructive interference?

- (1) 100 nm (2) 200 nm (3) 392 nm (4) 400 nm (5) 784 nm (6) 800 nm

(d) For what thickness of film will light of wavelength 560-nm (in air) exhibit destructive interference?

- (1) 100 nm (2) 200 nm (3) 392 nm (4) 400 nm (5) 784 nm (6) 800 nm
-

4. Light is normally incident from air ($n = 1.00$) onto a thin film of oil ($n = 1.40$) floating on an organic liquid ($n = 1.50$).

(a) What is the phase change of the ray reflected back into the air from the top of the oil film?

- (1) 0° (2) 180° (3) An angle between 0° and 180°

(b) What is the phase change of the ray reflected back into the oil from the bottom of the oil film?

- (1) 0° (2) 180° (3) An angle between 0° and 180°

(c) For what thickness of film will light of wavelength 560 nm (in air) exhibit constructive interference?

- (1) 100 nm (2) 200 nm (3) 392 nm (4) 400 nm (5) 784 nm (6) 800 nm

(d) For what thickness of film will light of wavelength 560-nm (in air) exhibit destructive interference?

- (1) 100 nm (2) 200 nm (3) 392 nm (4) 400 nm (5) 784 nm (6) 800 nm

5. An oil film on a water puddle usually has a whitish edge around it. What does this imply? Hint: at the edge of the slick the thickness is tapering off to zero.

14c. Polarization of light

The electric field vector of an electromagnetic wave is transverse to the direction of propagation of the wave. If the electric field vector remains parallel to a line in space, the wave is said to be linearly (or plane) polarized in the direction of that line.

Light from incandescent sources (like the sun) is randomly polarized. This light can be polarized by (1) selective absorption, (2) reflection, (3) scattering, and (4) birefringence or double refraction.

Unpolarized light can be converted into polarized light by:

(1) *Selective absorption*: the absorption of one direction of polarization by certain materials. Polaroid (invented by Edwin H. Land in 1938) contains long-chain hydrocarbon molecules which are made conducting (at optical frequencies) so that they absorb light energy for one direction of polarization, transmitting light polarized along the transmission axis. Half of unpolarized light is transmitted; a fraction $\cos^2 \theta$ is transmitted of light polarized at an angle θ relative to the transmission axis, a formula known as Malus' law.

(2) *Reflection*. When unpolarized light is reflected off a plane surface (such as water or glass), the reflected light is partially polarized. When the angle of incidence is such that the reflected and refracted rays are perpendicular to each other, the reflected light is completely polarized, a fact discovered in 1812 by Sir David Brewster. If this angle of incidence is θ_p , then

$$n_1 \sin \theta_p = n_2 \sin \theta_2 = n_2 \sin (90^\circ - \theta_p) = n_2 \cos \theta_p$$

so $\tan \theta_p = \frac{n_2}{n_1}$, an equation known as Brewster's Law.

For glass ($n_2 = 1.50$) in air ($n_1 = 1.00$) Brewster's angle is $\theta_p = \tan^{-1}(1.5) = 57^\circ$.

(3) *Scattering*. Light scattered perpendicular to its original direction of propagation will be linearly polarized in a direction perpendicular to the plane of the original and final directions of propagation. For example, unpolarized light traveling in the x direction and scattered in the z direction will be polarized along the y axis.

(4) *Birefringence or double refraction*. Some crystals (such as calcite) exhibit an ordinary type of refraction and an extraordinary type, the two beams being linearly polarized along perpendicular directions.

Example. Randomly polarized light passes through a polarizer oriented at $\theta = 30^\circ$ and then a second one oriented at $\theta = 75^\circ$. What fraction of the original light passes through each polarizer? Solution: Being randomly polarized, 50% of the light passes through the first polarizer (regardless of its orientation). The fraction of this which passes through the second polarizer, at an angle of $75^\circ - 30^\circ = 45^\circ$ with respect to the first, is $\cos^2 45^\circ = 0.50$, so a total of $50\% \times 0.50 = 25\%$ of the initial light makes its way through both polarizers.

If the light had originally been polarized at $\theta = 0^\circ$, the fractions of the initial light would have been $\cos^2 30^\circ = 0.75$ through the first and then $0.75 \times 0.50 = 0.375$ through the second.