Topic 7. Ray (Geometrical) Optics

Reflection and Refraction

Geometrical optics is that branch of optics which treats light as made up of rays that move in straight lines until they encounter an interface between two different materials, at which point they may reflect or refract, as described below. It is often referred to as “ray optics” because the light is treated as rays. Geometrical optics is to be distinguished from wave optics, in which the light is regarded as made up of waves. Geometrical optics applies when the dimensions of the system involved (like the distance across a lens) are many orders of magnitude greater than the wavelength of the light. Conversely, wave optics is necessary when this is not the case.

Topic 7 will consider just one form of electromagnetic wave: visible light. We will be examining the path traveled by light as it reflects off of surfaces, and as it propagates through certain types of (transparent) materials. Although everything we discuss here is valid in some form for other e-m waves, the particular materials which are relevant in this chapter (such as glass and water) would not be as appropriate for a similar discussion of, for instance, the paths traveled by radio waves. The propagation properties of e-m waves in various materials are strongly dependent on the frequency of the waves. In this chapter we focus on the particular behavior of light waves and their interaction with materials since light is of such great practical importance.

A concept that will be very important to us here is that of a "point source" of e-m waves. This is simply an emitter that is so small we can always ignore the detailed pattern of its emitted e-m radiation. We assume that all point sources emit e-m waves in all directions. The waves spread outward in a three-dimensional pattern, moving away from the source as do water waves from a rock dropped in a lake (or, more precisely, as do the waves from a firecracker exploding underneath the water's surface!). In the diagram below, we show a few of the electric field vectors contained in the e-m waves as they spread out from the point source at the center. We also show an observer (indicated by the "eye" at the right); the dashed cone indicates the path traveled by the small portion of the emitted light that strikes the observer's eye.

From the observer's point of view, all of the light that is seen has traveled along virtually the same straight-line path out from the source. An observer at any other point would see the same thing. The path traveled by the light from a point source to an observer is a straight line from the source, perpendicular to the electric and magnetic field vectors that compose the e-m wave. This path is called a "ray." In effect, the point source can be thought of as an emitter of
light along an infinite number of rays. We can think of a "light ray" as a narrow beam of light traveling in a straight line. And so, from now on, we will imagine that the point source emits "rays" of light outward in all directions, as illustrated here:

Now, any actual object that is emitting light (or reflecting light) is not actually a point source, but it does behave as if it were entirely composed of many point sources. That is, each point on an object emits an infinite number of light rays in all directions. It is essential to keep this idea in mind when analyzing optical phenomena!

To the right we show a typical "object" that is often used in optical experiments. It is simply an arrow, which may be drawn on a piece of paper or might actually be an "arrow-shaped" light source. We have picked out just two points on this arrow: one at the tip, and one in the middle of the tail. We have drawn just a few of the light rays that are emitted from these two points on the object. Light rays actually head out from all parts of the object, either because the object itself is producing the light or, more likely, the different parts of the object are reflecting light from some light source (such as the sun or a light bulb).

The image will then be drawn as another arrow, sometimes the same size as the object but more often larger or smaller, and not necessarily oriented in the same direction.

Before we can discuss images formed from the many light rays emitted by a luminous object, we need to analyze the path followed by a single ray of light as it travels from one material to another. That leads to our first question:

What happens when light traveling in one material encounters another material? As was discussed in Topic 11, the speed of all types of e-m waves in a vacuum is equal to $c$. However when traveling through material substances, the speed of e-m waves is less than $c$, and the precise speed will depend on the frequency of the particular e-m wave. In the context of light waves, the ratio between $c$ and the speed in the material is called "$n$," or the "index of refraction" of the material: $n_A = c/v_A$, where $v_A$ is the speed in material A, and $n_A$ is the index of refraction of material A. What happens, then, when light traveling in material #1 with index of refraction $n_1$ encounters a different material #2 that has index of refraction $n_2$? This
situation is represented in the diagram below (where the arrow indicates the path along which light is traveling):

![Diagram of light reflection and refraction]

The answer to this question, which is of great practical importance, can be provided by Maxwell's theory of electromagnetism. (The mathematical formulations of the basic principles of Maxwell's theory are called "Maxwell's equations.") Here we will just state the results without going into any of the mathematical derivations.

When light strikes an interface, some light is reflected and some is transmitted. In general, what happens when light encounters an interface between two different materials is this: some of the light goes on through into the new material, and some of it is "reflected" off of the interface and continues traveling in the first material along an altered path. The path of the reflected light, as well as that of the transmitted light, will depend on the precise path of the incoming, or "incident," ray. In order to specify all of the different directions involved, it is again convenient to draw a "normal" line perpendicular to the surface. We'll indicate that "normal" with a dashed line. (Note that all three rays are represented as being in the same plane [here, the plane of the page]. Maxwell's equations prove that this will always be true.)

![Diagram showing various angles and normals]

Here's some important terminology based on this diagram:

1. The incoming ray is called the "incident" ray. The reflected ray is simply referred to as the "reflected" ray. The transmitted ray (traveling in material #2) is called the "refracted" [bent] ray because, in general, it does not continue along the same path as the incident ray.

2. Angle $\theta_1$ is called the "angle of incidence." It is the angle between the incident ray and the normal to the surface.

Examples of $n$:
- $n_{air} = 1.0$
- $n_{water} = 1.3$
- $n_{glass} = 1.5$
- $n_{diamond} = 2.4$
(3) Angle $\theta_3$ is called the "angle of reflection." It is the angle between the reflected ray and the normal to the surface.

(4) Angle $\theta_2$ is called the "angle of refraction." It is the angle between the refracted ray and the normal to the surface.

**THE LAW OF REFLECTION**

The diagram at the bottom of the previous page shows reflection of light off a smooth reflecting surface. The reflected ray obeys the following simple and straightforward law:

**Law of Reflection:** When light reflects from a surface, (1) the reflected ray lies in the plane of incidence determined by the incident ray and the normal to the surface, (2) the reflected ray is on the other side of the normal from the incident ray, and (3) the angle of reflection $\theta_3$ equals the angle of incidence $\theta_1$.

**THE LAW OF REFRACTION**

The transmitted (refracted) ray obeys the following law:

**Law of Refraction:** When light refracts through an interface between two different materials, (1) the refracted ray lies in the plane of incidence determined by the incident ray and the normal to the surface, (2) the refracted ray is on the other side of the normal from the incident ray, and (3) the angle of refraction $\theta_2$ is related to the angle of incidence $\theta_1$ by Snell's Law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, where $n_1$ and $n_2$, respectively, are the indexes of refraction in the initial and final materials.

Snell's Law indicates that if the incident light is refracted into a material with a higher index of refraction, $\theta_2 < \theta_1$, then the transmitted ray is bent towards the normal, as shown below in the diagram on the left. If the incident light is refracted into a material with a lower index of refraction, $\theta_2 > \theta_1$, then the transmitted ray is bent away from the normal, as shown in the diagram on the right.

Typically, one knows the values of the index of refraction on the two sides of the interface, and the angle of incidence. Then Snell's law is used to determine the angle of refraction:

$$\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} \quad \text{or} \quad \theta_2 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_2} \right).$$
If \( n_1 < n_2 \) then \( \theta_2 < \theta_1 \), meaning that the refracted ray has been bent back towards the normal, as shown in the diagram on the left. There is always a solution to Snell's law in this case.

If \( n_1 > n_2 \) then \( \theta_2 > \theta_1 \), and the refracted ray has been bent away from the normal. In this case there might be no solution to Snell's law if \( \theta_1 \) is too large: Since \( \sin \theta_2 \) cannot be greater than 1, there is no solution to Snell's law if

\[
\frac{n_1 \sin \theta_1}{n_2} > 1 \text{ or } \sin \theta_1 > \frac{n_1}{n_2} \text{ or } \theta_1 > \sin^{-1}(n_1/n_2).
\]

So what happens in this case? There is no refracted ray! Only reflection can occur, and all the light is reflected back, and stays in, the medium from which the incident ray came. This phenomenon is called total internal reflection. In the case where the light is traveling into a material with lower index of refraction (for instance, from glass into air), it is possible to find an angle of incidence large enough that the angle of refraction is exactly 90°. For this angle of incidence, and any angle that is larger than it, the transmitted wave completely disappears. All of the incoming light will be reflected, and so this situation is called "total internal reflection." It is as if the light were "trapped" inside the material with the higher refractive index. The angle of incidence which results in a 90° angle of refraction is called the "critical angle" and denoted \( \theta_c \). Note that the critical angle depends on the refractive indices of both materials! If we assume again that \( n_2 > n_1 \), and that the light is traveling from \( n_2 \) into \( n_1 \), as shown in the diagram on the right at the bottom of the previous page, we can find the critical angle this way:

\[
n_1 \sin 90° = n_2 \sin \theta_c \quad \text{so} \quad n_1 = n_2 \sin \theta_c \quad \text{and} \quad \frac{n_1}{n_2} = \sin \theta_c
\]

so

\[
\theta_c = \sin^{-1}(n_1/n_2)
\]

Total internal reflection is the basis of fiber optics. Optical fibers are long, thin glass tubes. Light beams are directed into one end and continue on through the entire fiber while losing essentially no energy, finally exiting the other end. Each time the light beam strikes an internal surface of the fiber, it is totally internally reflected and so no light leaves the tube until the very end. These light beams (usually produced by lasers) are used to carry voice and data signals with very high efficiency.
Total internal reflection also explains the color and sparkle of diamonds. The index of refraction of diamond is 2.4 - very nearly the highest of any substance. Because of the high index of refraction, the critical angle for diamond is very low: only around 24°. When light enters one face of the diamond and strikes an internal surface, it is very likely to have an incidence angle larger than that, in which case it will be totally internally reflected. Then, it strikes another internal face - and so on. It is almost as if a collection of mirrors were trapped inside the diamond. Whatever the angle from which the diamond is viewed, an observer is likely to see one of the light beams that has been bouncing around inside. In addition, as the light beams travel inside the diamond, light of different colors follow slightly different paths. (This is due to the fact that the indices of refraction for different frequencies of light have slightly different values.) By the time the light exits the diamond, it has "spread out" into a small rainbow of different colors. Because of its great hardness, diamond can be cut into intricate shapes that enhance these effects.

Some nice demonstrations of total internal reflection will be shown in class. Watch for the one with red laser light following a stream of water from a large transparent box into the sink.

**Example 1.** Suppose light is incident from air \((n = 1.00)\) onto water \((n = 1.33)\) at an angle of incidence of 30°. Determine the angles of reflection and refraction and draw a diagram showing the incident, reflected, and refracted rays.

Solution: The angle of reflection must also be 30°. The angle of refraction is found by Snell's law: \(1.00 \sin 30° = 1.33 \sin \theta_2\) so

\[
\theta_2 = \sin^{-1}[1.00 \sin 30°/1.33] = \sin^{-1}[1.00 \times 0.500/1.33] = \sin^{-1}[0.375] = 22°.
\]

Considering the precision of the original numbers, determining this angle to the nearest degree seems adequate.
Example 2. Imagine a piece of glass \((n = 1.60)\) with flat parallel sides immersed in water \((n = 1.33)\) on both sides. Light is incident (from the water above the glass plate) onto the glass at an angle of incidence of 45°. The light that is refracted into the glass also reflects and refracts at the bottom surface of the glass. Determine the angles of reflection and refraction at both the top and bottom surfaces of the glass, and sketch the reflected and refracted rays. Also show what happens (for several reflections and refractions) when the ray reflected from the bottom of the glass returns to the top surface.

Solution: The angle of reflection at the top surface must also be 45°. The angle of refraction at the top surface is found by Snell's law: \(\sin \theta_2 = \frac{\sin \theta_1}{n_2} = \frac{\sin 45°}{1.60}\) so

\[
\theta_2 = \sin^{-1}\left(\frac{\sin 45°}{1.60}\right) = \sin^{-1}\left(\frac{0.707}{1.60}\right) = \sin^{-1}(0.588) = 36°.
\]

QUESTIONS

Light is incident from air \((n = 1.00)\) onto a piece of glass \((n = 1.60)\).

1. If the angle of incidence is 32°, the angle that the reflected ray makes with the normal is

   (1) nonexistent  (2) 19°  (3) 37°  (4) 41°  (5) 58°

2. If the angle of refraction is 32°, the angle of incidence was

   (1) nonexistent  (2) 19°  (3) 32°  (4) 41°  (5) 58°

3. If the light is incident from the glass, what is the critical angle for total internal reflection?

   (1) nonexistent  (2) 19°  (3) 29°  (4) 39°  (5) 59°
4. When light is incident from air \((n = 1.00)\) onto an organic liquid, it is found that if the angle of incidence is \(45^\circ\), the angle of refraction is \(30^\circ\). What is the index of refraction of the liquid?

| (1) 1.11 | (2) 1.21 | (3) 1.31 | (4) 1.41 | (5) 1.51 |

5. Light is incident from water \((n = 1.33)\) onto a flat piece of glass of index of refraction 1.66. Determine the angle of refraction for angles of incidence of \(45^\circ\) and \(60^\circ\) and make sketches of the incident, reflected, and refracted rays.

\[
\begin{array}{ll}
n = 1.33 & n = 1.33 \\
n = 1.66 & n = 1.66 \\
\end{array}
\]

6. Light is incident from a flat piece of glass of index of refraction 1.66 into water \((n = 1.33)\). Determine the angle of refraction for angles of incidence of \(45^\circ\) and \(60^\circ\) and make sketches of the incident, reflected, and refracted rays.

\[
\begin{array}{ll}
n = 1.66 & n = 1.66 \\
n = 1.33 & n = 1.33 \\
\end{array}
\]

7. A transparent organic liquid of unknown index of refraction floats on top of a piece of glass \((n = 1.55)\). It is found that total internal reflection occurs when light is incident from the liquid at an angle of incidence of \(68^\circ\). What is the index of refraction of the liquid?

| (1) 1.32 | (2) 1.44 | (3) 1.55 | (4) 1.67 | (5) 1.79 |
IMAGES

**Formation of an "image" by many light rays.** The most important practical applications of optical principles are associated with the formation of images. Images of objects can be created by carefully designing systems to redirect the paths of light rays, making use of reflection (with mirrors) and refraction (with lenses).

We may describe an "image" of an object as a sort of "map" - a map composed of light rays, formed from an object that is emitting or reflecting light. We call it a "map" because each point on the object corresponds to a point on the image, and some of the geometrical "shape" relationships of the object are retained in the map. But, like any map, there may also be some distortion of the original object. It may be magnified or reduced, turned upside down, inverted "left-to-right," or have its shape changed in other ways. Images are formed by microscopes, telescopes, cameras, projectors, and many other devices. And, of course, there is another extremely important use of images: the images formed on our retinas by the lenses in our eyes!

The principles behind all of these methods of forming images are very similar. The image is formed at a specific location, e.g. on the retina, on a screen, on a sheet of film, etc. Light rays from the original object are redirected to the image location, either by reflection from mirror-like surfaces or refraction through transparent materials. A lens is a piece of transparent material shaped so as to form images through refraction. Here we will discuss some of the basic principles behind the operation of a lens.

**MIRRORS**

A mirror is a device with a highly-reflecting surface. Its properties can be determined using just the laws of reflection. **Plane mirrors** have plane reflecting surfaces and **spherical mirrors** have mirrors that are a part of a large sphere. (There are also other types of mirrors, such as parabolic mirrors.) The plane mirror is the simplest type of mirror, and a study of a plane mirror exhibits many of the important properties of a more complicated mirror, so we will study it first.

The diagram on the left on the next page is a plan view which shows a physical situation as viewed from directly above. It shows a number of different rays of light emerging from a point object at the point labeled O (representing an "Object") and heading in different directions. Nearby is a plane mirror indicated on the diagram. Some of the rays do not strike the mirror while others do. The ones that do strike the mirror then reflect off the mirror in accordance with the law of reflection. If we dash in the extensions of these reflected rays onto the far side of the mirror, it will be observed that all of them pass through the point I (representing an "Image") which is located directly opposite the object, and the same distance behind the mirror as the object is in front of the mirror. An observer at location P would see light directly from the object O and light from O that first reflected off the mirror and now appears to be coming from the point I instead. Since light does not really originate from I, but only appears to originate there, this image is referred to as a "virtual" image. Depending on where an observer stands and what obstructions are in the way, the object only might be visible, or the image only, or both.
The diagram on the right shows the same physical situation from the side. The image is seen to be "erect," that is to say, it is oriented in the same direction as the object: the top of the image is the image of the top of the object, and the bottom of the image is the image of the bottom of the object. It is also clearly the same size and color as the object: if the object is, say, five-feet-two, eyes-of-blue, then the image will also be five-feet-two, eyes-of-blue.

**Left and right, front and back.** If you look at your image in a mirror, your image is clearly right-side up (erect). However, if you wave your right hand, your image appears to wave its left hand. How can a plane mirror reverse left and right but not up and down? The fact of the matter is that a plane mirror only reverses front and back, and this has the effect of appearing to reverse left and right. However, the image of your right hand is actually still on your right.

**CHARACTERISTICS OF IMAGES**

Image formation occurs when the rays from an object, after reflection or refraction, particularly at curved interfaces, pass through a point or appear to come from a point. Images can be classified as: (1) real or virtual, (2) erect and inverted, (3) larger or smaller or the same size as the object.

An image is:

1. **Real** if the light rays actually do pass through the image point or **virtual** if they do not, but only appear to have passed through the image point.

   In the case of a lens, light rays from an object are transmitted through a lens to the other side, and if the image is formed on that side it is a real image. Otherwise, if the image is on the same light as the incident light ray, the image is virtual, and the rays only appear to come from that point.

   In the case of a mirror, light rays reflected off the mirror return to the side from which they cam, and if the image is formed on that side it is a real image. Otherwise, if the image is on the opposite side (the back side of the mirror), the image is virtual, and the rays only appear to come from that point.

2. **Erect** if the image has the same orientation as the object or **inverted** if the image is inverted with respect to the object.
3. Larger or smaller than the object; the relative linear dimensions of the image and object are referred to as the **magnification** \( m \). If an object is 3.00 mm high and its image is 1.50 mm high, we say the magnification is

\[
m = \frac{\text{image height}}{\text{object height}} = \frac{1.50 \text{ mm}}{3.00 \text{ mm}} = 0.50.
\]

Note that magnifications may be greater or less than 1. By convention, a positive magnification refers to an erect image and a negative magnification to an inverted image. Thus a magnification of +0.5 would mean that the image is erect and half as tall as the object, while a magnification of −3.0 would mean that the image is inverted and three times as tall as the object.

The formula for magnification (for either mirrors or lenses) is

\[
m = -\frac{i}{o},
\]

where \( i \) and \( o \) are, respectively, the object and image distances from the interface, discussed below. This formula leads to the correct magnitude and sign for the magnification, if you remember to include the − sign!

**DIFFERENT TYPES OF MIRRORS**

**The Concave Mirror with \( o > f \)**

The diagram on the right shows the image formation for a mirror and defines some important quantities associated with mirror optics. This mirror is a **spherical mirror**, meaning a part of a spherical surface. The center of that sphere, called the **center of curvature** (marked C on the diagram), is on the left side for this particular mirror, and the reflecting surface will be the left side of the mirror.

The long line passing through the center of curvature and the middle of the mirror is called the **principal axis of the mirror**. The point at which the principal axis meets the mirror is called the **vertex** of the mirror; it is marked V. We will only consider objects placed close to the principal axis in our study of mirrors and lenses.

The object could be any small object, but is conventionally represented as an arrow located on the principal axis. The head of the arrow indicates the orientation of the object.

This type of mirror — with its center of curvature on the same side as the object — is called a **concave mirror**.

Two rays are shown emanating from the head of the arrow. As they strike the mirror, they are reflected according to the law of reflection. One ray passes from the object through the center of curvature, and must necessarily strike the mirror at an angle of incidence of 0° (because it is along a radius of the mirror and strikes the mirror perpendicularly); this means it reflects straight back along the same line. The other ray strikes the vertex of the mirror, and must reflect back on the opposite side of the principal axis at exactly the same angle with the principal axis as the incident ray.
Note that the two reflected rays cross at a certain point on the lower side of the principal axis. That point is the image of the head of the arrow marking the object. The image can then be drawn at that location, starting from the principal axis.

The distance from the object to the mirror (measured along the principal axis) is called the object distance and denoted by the symbol \( o \). The distance from the image to the mirror (also measured along the principal axis) is called the image distance and denoted by the symbol \( i \). In this case both \( o \) and \( i \) are given positive values, because they are on the “real” side of the mirror, the side on which both the incident and reflected rays are “really” found.

If a number of different object distances are used, and their image distances determined, it will be found that \( 1/o + 1/i \) always has the same value. We write this relationship as

\[
1/o + 1/i = 1/f.
\]

The quantity \( f \) is called the focal length of the mirror. For the concave mirror shown in the diagram, it is found that \( f = R/2 \), where \( R \) is the radius of the spherical mirror (the distance from the center of curvature to the mirror). What is the meaning of the focal length? It is the image distance for a very distant object. A very distant object has \( o \approx \infty \) or \( 1/o \approx 0 \) so that \( 1/i = 1/f \) and \( i = f \).

It is also found that \( m = -i/o \) gives the correct magnification (including sign: + for an erect image and − for an inverted image) of the image.

We will now define one more point associated with the mirror: the focal point is the point halfway between the vertex of the mirror and its center of curvature. The distance from the mirror to the focal point is then \( f = R/2 \).

Since rays from a very distant object must be parallel to the principal axis, they will reflect through the focal point. And rays that come from the object and pass through the focal point must reflect back parallel to the principal axis. This gives us now a total of four different rays that can be drawn to locate the image formed by a concave mirror. These four rays are shown on the diagram to the right.

Use two or more of them when asked to draw ray diagrams in homework problems or on exams.

**The Concave Mirror with \( o < f \)**

Everything so far has been drawn for the case where the object is located farther from the mirror than the focal point. When this is the case, the image is always a real, inverted image. The situation is very different when the object is located closer to the mirror than the focal point, as in the diagram below. In this case three of the rays used before can be drawn in a similar fashion, but one is different: the ray that originally passed from the object through
the focal point and then reflected from the mirror parallel to the principal axis needs to be drawn so that it appears to come from the focal point through the object, and then reflects back parallel to the principal axis. On the diagram, a dotted line is used from the focal point to the object to indicate this.

The rays in this case do not converge to a real image point on the left side of the mirror. Instead, it is necessary to dash in extensions of the reflected rays (not extensions of the incident rays) back to the right side to locate the image. The reflected rays on the left side appear to come from the virtual image on the far side of the mirror. Of course, there are no rays of any kind on the right side of the mirror, the reflected rays only appear to come from an image there.

In this case the image is said to be virtual and erect. This can be seen not only from the ray diagram, but also from the mirror equation: In a calculation using the mirror equation $1/o + 1/i = 1/f$ for a concave mirror, an object closer to the mirror than $f$ means that $o < f$ so that $1/o > 1/f$ so $1/i$, and therefore $i$, must be negative. A negative $i$ implies a virtual image, and the magnification $m = -i/o$ will then be positive, implying an erect image.

**Summary of Ray Tracing for Concave Mirrors**

The location and characteristics of the image of a concave spherical mirror can be determined by ray tracing methods using two or more of these rays:

1. A ray along a radial line (a line passing through the center of curvature) reverses its path upon reflection.
2. A ray parallel to the axis reflects back through the focal point.
3. A ray through the focal point (or imagined as coming from the focal point) is reflected back parallel to the principal axis.
4. A ray incident at the vertex of the mirror reflects back on the other side of the principal axis at the same angle.

Normally, draw the rays from the head of the object. All the rays should intersect at the same point, which is the head of the image. While two rays are normally sufficient to locate an image, it is better to use at least three to make sure you have the correct location.
**The Convex Mirror**

Next, let us consider the case of a **convex mirror**, which is a mirror with the center of curvature on the back side of the mirror, the virtual side on which no light rays are found. The bottom of a shiny metal spoon is a convex mirror, and outside rear view mirrors and security mirrors are normally also convex mirrors. Again, as for a concave mirror, there is a focal point (also on the virtual side) halfway between the vertex and the center of curvature. The focal length is then taken to be negative, so \( f = -\frac{R}{2} \) (instead of \( f = +\frac{R}{2} \) for the concave mirror).

From the mirror equation \( \frac{1}{o} + \frac{1}{i} = \frac{1}{f} \) we can derive the equation \( \frac{1}{i} = \frac{1}{f} - \frac{1}{o} \), from which we see (since \( f \) and thus \( \frac{1}{f} \) are negative), that \( \frac{1}{i} \) and thus \( i \) must be negative. A convex mirror thus always has virtual images (for real objects), and those images are erect.

The image can also be located using rays. The same four rays used for the concave mirror can also be drawn for the convex mirror, but they have to be treated a little differently since the focal point and center of curvature are on the extension of the principal axis on the back side of the mirror. The diagram below shows the four rays for a convex mirror. The rays are:

1. A ray headed towards the center of curvature \( C \) reverses its path upon reflection.
2. A ray parallel to the axis reflects back as though it came from the focal point \( F \).
3. A ray headed towards the focal point \( F \) is reflected back parallel to the principal axis.
4. A ray incident at the vertex of the mirror reflects back on the other side of the principal axis at the same angle.

The diagram below shows ray tracing for the location of the image of a convex mirror.

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**Outside mirrors on automobiles.** The outside rear-view mirrors on cars are sometimes plane mirrors and sometimes convex mirrors. The convex mirrors often carry a warning to the effect that objects seen in this mirror are closer than they appear to be. Why? Think about the magnification of a convex mirror: the image is virtual, erect, and smaller than the object. The image of an approaching car is really what you see, not the car itself. Consequently, the car viewed in the mirror would appear to be smaller than it really is, possibly causing the driver to think the car is farther away than it really is.
The plane mirror

Let's now revisit the plane mirror. The surface of a plane mirror is a plane, rather than part of a sphere. However, it can be considered to be a spherical mirror of infinite radius of curvature, and thus infinite focal distance. The mirror equation for a plane mirror can be written with \(1/f\) replaced by 0, so it just becomes:

\[
\frac{1}{o} + \frac{1}{i} = 0 \quad \text{which implies} \quad \frac{1}{i} = -\frac{1}{o}
\]

which implies \(i = -o\) and \(m = -i/o = +1\).

Thus a plane mirror produces a virtual image at the same distance behind the mirror as the object is in front of the mirror, and the image is erect (because \(m\) is positive) and exactly as large as the object. This agrees with reality, doesn't it?

Example 1: You are standing five feet from a long plane mirror, looking straight ahead. Another person is standing 5 feet to your right but two feet behind you (in other words, 7 feet from the mirror). What part of the mirror allows you to “see” what that person is doing? Sketch a diagram to show your answer.

You need only a small part of the mirror, the part that exactly reflects light from the other person towards you. It is on the straight line from you to the image of the other person.

How far has the light traveled in its path from the other person to you?

Same as the distance from you to the image of the other person:

\[
\sqrt{(5\ f)^2 + (12\ f)^2} = \sqrt{25 + 144} \ f = \sqrt{169} \ f = 13 \text{ feet.}
\]

A “full-length” mirror. A full-length mirror, one which you can use to see your image from head to toe, only needs to be half your height, with its bottom halfway between the bottom of your feet and your eyes and the top halfway between your eyes and the top of your head. To see this, draw a diagram from the side which shows yourself, the mirror, and your image, and shows the path the rays really follow to allow your eyes to see the image of the top of your head and the bottom of your feet.
Image Location for Spherical Mirrors Using the Mirror Formula

Ray tracing is a geometrical method of locating and describing the images formed by mirrors. The same thing can be done mathematically using the mirror formula $1/o + 1/i = 1/f$, where $o$ is the object distance, $i$ is the image distance, and $f$ is the focal length. This formula applies to both mirrors and lenses, but there are some subtle differences in how they are applied.

A spherical mirror can be concave or convex. A concave mirror curves towards the subject, while a convex mirror curves away from it. If you hold a metal spoon and look into the top side (the bowl) of the spoon, you are looking into a concave mirror. If you look at the bottom side, you are looking into a convex mirror. For a concave mirror, the radius of curvature $R$ is taken to be positive because the center of curvature is on the same side as the incident light. For a convex mirror, $R$ is taken to be negative because the center of curvature is on the opposite (virtual) side: $R < 0$.

A concave mirror (which curves towards the object) has a positive $R$ and has a focal length $f > 0$. The image of a real object formed by a concave mirror is erect and virtual and enlarged if the object is inside the focal point but inverted and real if the object is outside the focal point. In other words, if $o < f$, the equation above requires that $i < 0$, but if $o > f$, it requires that $i > 0$.

A convex mirror curves away from the object and has a focal length $f < 0$; its image of a real object is always virtual, erect, with magnification $0 < m < 1$.

The side of a mirror on which a real object is located is referred to as the positive side of the mirror, whichever side it is, and we take $o > 0$. For a concave mirror, where the center of curvature and the focal point are also located on the positive side, the focal length $f = R/2$ is positive. For a convex mirror, where the center of curvature and the focal point are on the opposite (negative) side of the mirror, $f = -R/2$.

If $i > 0$, the image will be on the positive side and it will be a real image, meaning if you place a screen there you will actually see the image at that location. If $i < 0$, the image will be on the negative side and it will be a virtual image; a screen at that location won't show anything, but the reflected rays from the real object appear to come from that position.

The linear magnification of the image can be determined from the formula $m = -i/o$, where $i$ and $o$ are the image and object distances (keep their proper signs!). The sign of $m$ tells whether the image is erect (same orientation as the object) or inverted (turned upside down relative to the object); positive $m$ means an erect image and negative $m$ means an inverted image.
Note that for a convex mirror, for which $f$ is negative, $i$ must also be negative according to the formula, so we know that all images from a convex mirror are virtual (because $i$ is negative), erect (because $m$ will be positive), and smaller than the object (check and you'll see that $|i|$ will always have to be less than $|o|$ in this case).

For a concave mirror, the image might be real and inverted or it might be virtual and erect, depending on where the object is located.

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**Example 2:** If the object is located closer to a concave mirror than its focal point, what type of image will occur?

If $o < f$ then $1/o > 1/f$ so $1/i$ must be negative so $i$ is also negative. The image is then virtual, erect, and smaller than the object.

**Example 3:** If the object is located farther from a concave mirror than its focal point, what type of image will occur?

If $o > f$ then $1/o < 1/f$ so $1/i$ must be positive so $i$ is also positive but less than $f$. The image is then real, inverted, and smaller than the object.

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**QUESTIONS**

1. An object is located 15 cm from a plane mirror. Determine the object distance, image distance, and magnification for this situation.

2. A concave spherical mirror has a radius of curvature of 20 cm. (a) Determine its focal length. Also, for an object located 15 cm from the mirror, determine its object distance, image distance, and magnification, and characterize the image (real/virtual, larger/smaller/same size as the object, erected or inverted).
(b) Sketch out a diagram for this situation and use ray tracing to locate and characterize the image, then check with your previous answers.

3. A different concave spherical mirror has a radius of curvature of 12 cm. (a) Determine its focal length. Also, for an object located 4 cm from the mirror, determine its object distance, image distance, and magnification, and characterize the image (real/virtual, larger/smaller/same size as the object, erected or inverted).

(b) Sketch out a diagram for this situation and use ray tracing to locate and characterize the image, then check with your previous answers.
4. A convex spherical mirror has a radius of curvature of 20 cm. (a) Determine its focal length. Also, for an object located 10 cm from the mirror, determine its object distance, image distance, and magnification, and characterize the image (real/virtual, larger/smaller/same size as the object, erected or inverted).

(b) Sketch out a diagram for this situation and use ray tracing to locate and characterize the image, then check with your previous answers.
LENSES

A simple lens is a single piece of glass (or clear plastic) with at least one curved surface, through which light is allowed to pass from one side to the other, with refraction of the light taking place at each surface. More complex lenses may have several pieces of glass, either glued to one another or placed very close to each other.

The properties of a simple lens are determined by (1) the index of refraction of the glass and (2) the shapes and distances apart of the two surfaces. Although the index of refraction varies with the frequency of the light (or other electromagnetic wave), we will usually assume that it is constant.

Lenses are placed into two groups, convergent and divergent, according to whether they are thicker or thinner in the middle than at the top and bottom of the lens:

(1) **Convergent lenses** are thicker in the middle than at the top and bottom, and tend to make incident rays converge towards the principal axis; they have a positive focal length $f > 0$. Some examples of convergent lenses are shown to the right.

(2) **Divergent lenses** are thinner in the middle than at the top and bottom, and tend to make incident rays diverge away from the principal axis; they have a negative focal length $f < 0$. Some examples are shown to the right.

The focal length of a lens is not as easily calculated as the focal length of a mirror, so we will simply assume that there is a focal length $f$. Sometimes $f$ will be given, sometimes it can be inferred from other information. While a mirror has only one focal point (the point midway between the center of curvature and the vertex of the mirror), a lens has two focal points, one on either side of the lens.

As with mirrors, images can be located and characterized either by using the lens equation $1/o + 1/i = 1/f$ and the magnification equation $m = -i/o$, or by ray tracing. We'll begin with ray tracing (which varies a little between convergent and divergent lenses) and then consider the analytical results.
IMAGE LOCATION USING RAY TRACING

The image of a lens can determined by ray tracing methods. Useful rays are the following:

1. A ray from a point on the object to the center of the lens passes straight on through, regardless of whether the lens is convergent or divergent.

2. A ray from a point on the object parallel to principal axis continues through the focal point on the other side in the case of a convergent lens, or diverges from the focal point on the object side in the case of a divergent lens.

3. A ray directed toward or away from a focal point emerges parallel to the principal axis. For a convergent lens, use the focal point on the same side as the object; for a divergent lens, use the focal point on the opposite side.

Check that except for ray (1), the rays tend to converge (come closer to the principal axis) for a convergent lens and diverge for a divergent lens. If not, you drew something wrong!

For a real object, a divergent lens will always have a virtual image located on the same side as the object. A convergent lens may have a real image on the other side of the lens, or a virtual image on the same side as the object, depending on where the object is located relative to the focal point.

Here is an example of a convergent lens. The object is on the right, at a positive object distance $o$. One focal point is shown, and two rays are traced to locate the image, which is at a positive image distance $i$ from the lens. Suggestion: Locate the other focal point of this lens and trace a ray through it and show that it intersects with the other two rays.

Here is an example of a divergent lens. The object is on the right, at a positive object distance $o$. One focal point is shown, and two rays are traced to locate the image, which is at a negative image distance $i$ from the lens. Suggestion: Locate the other focal point of this lens and trace a ray involving and show that its extension backwards intersects with the other two rays.
**IMAGE LOCATION USING THE LENS EQUATION**

The other method for locating and describing images is to use the formula \[ \frac{1}{o} + \frac{1}{i} = \frac{1}{f}, \] where \( o \) is the object distance, \( i \) is the image distance, and \( f \) is the focal length. This formula applies to both mirrors and lenses, but there are some subtle differences in how they are applied.

The object is located on one side of the lens (it doesn't matter which one), and \( o \) is positive. (Can \( o \) be negative? Only if it is actually the image from another lens or mirror, but the lens has intervened in the path of the light rays before the image was formed.)

For a convergent lens (one that is thicker in the middle than at the edges), the focal length \( f \) is positive. For a divergent lens (one that is thinner in the middle than at the edges), the focal length \( f \) is negative.

The image distance \( i \) is positive if the image is on the opposite side to the object (that's the side to which the light rays really go), and negative if it is on the same side as the object. A positive \( i \) corresponds to a real image, as it did for mirrors, and a negative \( i \) corresponds to a virtual image (again meaning that a screen at that location won't show anything, but the refracted rays from the real object appear to come from that position).

The linear magnification of the image can be determined from the same formula as for mirrors: \[ m = -\frac{i}{o}, \] where \( i \) and \( o \) are the image and object distances (keep their proper signs!). Again, the sign of \( m \) tells whether the image is erect (same orientation as the object) or inverted (turned upside down relative to the object); positive \( m \) means an erect image and negative \( m \) means an inverted image.

Note that for a divergent lens, for which \( f \) is negative, \( i \) must also be negative according to the formula, so we know that all images produced by a divergent lens are virtual (because \( i \) is negative), erect (because \( m \) will be positive), and smaller than the object (check and you'll see that \( |i| \) will always have to be less than \( |o| \) in this case).

For a convergent lens, the image might be real and inverted or it might be virtual and erect, depending on where the object is located.
COMMON OPTICAL INSTRUMENTS

Magnifier: A magnifier (magnifying glass) is a short-focal length convergent lens, perhaps \( f = 5 \text{ cm} \) or so, used to create a large virtual image of a small object. The lens is brought up close to the object, which is placed just inside the focal point of the lens; the virtual image is erect, with the same orientation as the object (so you don't need to turn it upside down). In theory, the lens can be used to create a large real image of the small object, by placing the object just outside the focal point of the lens, but this is usually not as convenient.

Slide Projector: The purpose of the slide projector is to project a large real image of a real object (the slide) onto a wall or screen, to make it visible to a large audience. A real image \((i > 0)\) is needed, and a large magnitude of the magnification \(m\). A short-focal length convergent lens can do this, with the object (the slide) placed just outside the focal point of the lens.

A Simple Fixed-Focus Film Camera: A simple camera (like the old "box" cameras and some of the cheaper disposable cameras available today) has a convergent lens which forms a real image on the surface of your film. The distance from the lens to the film is fixed in the simplest cameras. It is usually the image distance for an object about 5 m or 15 feet in front of the camera, and the image on the film will be acceptably sharp for objects from about 2 m or 7 feet to infinity, but a little out-of-focus for nearer objects.

A Simple Adjustable-Focus Film Camera: In this camera the lens can be moved out to increase the image distance (the distance from the lens to the film) and thereby focus on closer objects. When the lens is all the way back, the focal point of the lens coincides with the film, and the camera focuses on infinitely distant objects.

A Complex Camera: A more complex camera has many adjustments, such as the width of the aperture of the lens that is allowed to transfer light to the film; a wider aperture transfers more light, allowing a faster shutter speed and/or pictures under dimmer conditions. It may also have interchangeable lenses of different focal lengths, so you can use a 21-mm focal length wide-angle lens, a 50 mm "normal" lens, and a 200-mm telephoto lens. Also available are "zoom" lenses may have several pieces of glass, often with changeable inter-lens distances, allowing the focal length to be varied. A "fixed focal-length" lens might have \( f = 50 \text{ mm} \), while a zoom lens might range from \( f = 35 \text{ mm} \) (slightly wide angle) to \( f = 150 \text{ mm} \) (a telephoto lens).

Digital Cameras: A digital camera also has a lens, which might be a single-focal-length lens or a zoom lens. My Konica KD-5000Z has an 8-24 mm lens, which means a 3 to 1 optical zoom lens. This lens focuses its image on the digital sensor, which is much smaller than the film area on which a 35-mm camera produces its image.

The Eye: Your eye consists of a convergent lens at the front of the eyeball and the retina in the back of the eye, a distance \( D \) behind the lens, where a real image is formed and transmitted to your brain; \( D \) is then the image distance. In a sense, the eye is the original "digital camera." The lens of the eye does not focus in the usual way (which is by moving the "screen" relative to the lens), but by adjusting the focal length of the lens so the image distance remains \( D \); this is called "accommodation."
**Eyeglasses:** Myopic or near-sighted persons have lenses which can’t be relaxed enough to focus on distant objects, so that the image is formed inside the eyeball instead of on the retina; corrective divergent lenses can correct for this, enabling them to see out to infinity (but no longer as close as they could focus without eyeglasses). Hyperopic or far-sighted persons have lenses which can’t be made convergent enough to focus properly on nearby objects, so that the light reaches the retina before forming an image; corrective convergent ("reading") glasses can correct for this, but may interfere with focusing on distant objects. Convergent corrective lenses are also used by older persons who have become presbyopic, which means that the lens in their eye has lost its accommodation by becoming less flexible. Bifocals and trifocals allow different-focal-length parts of the eyeglasses to be used for objects at different distances.

**Example:** A near-sighted person can focus on objects in the range of 10 cm (4 inches) to 25 cm (10 inches), referred to as the "near point" and "far point." The person wants to be able to see objects out to infinity. What focal length eyeglasses would give an image at 25 cm of an object that is actually at infinity? This means find \( f \) which for \( o = \infty \) gives \( i = -25 \) cm (this will have to be a virtual image out beyond the eyeglasses). Answer: \( f = -25 \) cm (these are divergent lenses). Now for \( i = -10 \) cm (the person's near point) the corresponding \( o = 16 \) cm, so with eyeglasses this person can still see very close.

**Microscope:** A microscope creates a magnified image of a small object, but is intended to magnify much more than a simple magnifier would. This is generally accomplished by using two short-focal-lengths convergent lenses a short distance apart. Each lens produces a magnification greater than 1, and the total magnification is the product of the two magnifications.

**Refracting Telescope:** One purpose of a telescope is to gather more light than your eye would normally gather, in order to allow viewing of dim objects such as dim stars or galaxies; this is accomplished by using large-diameter lenses. The other is to magnify the objects in the field of view. This is accomplished by using two convergent lenses. One, the "objective," is a long-focal-length lens, and the other, the "eyepiece," is a short-focal-length lens placed near the focal point of the objective lens. A telescope owner usually pays for a good, large objective lens, and buys a variety of eyepieces to use to produce different magnifications. This type of telescope produces an inverted image, but astronomers rarely care whether an object is upside up or upside down. An erect image can be produced by using a divergent eyepiece; this type of telescope is often referred to as a **spyglass**.

**Reflecting Telescope:** A reflecting telescope uses a spherical mirror (or several mirrors) instead of an objective lens. The eyepiece is still normally a convergent lens.

**Binoculars:** A pair of binoculars is just a matched pair of telescopes, one for each eye. A good pair should allow individual focusing for each eye. Binoculars are kept short (compared to a telescope) by "folding" the path, usually using reflecting prisms visible on the binoculars. Binoculars differ in their light-gathering ability (large lenses are better for dim light, such as nighttime use) and their "power" (magnification), as well as their field of view (the angle between the left and right sides of the viewed region).
QUESTIONS

For each of the following, draw a diagram that is approximately to scale and use it to trace out rays that enable you to locate and characterize the image. Then use the lens equation to check your answer.

(1) A convergent lens with $f = 10$ cm, with the object located at $o = 15$ cm.

(2) A convergent lens with $f = 20$ cm, with the object located at $o = 15$ cm.

(3) A divergent lens with $f = 20$ cm and $o = 15$ cm.

(4) A divergent lens with $f = 10$ cm and $o = 15$ cm.