TOPIC 1. REVIEW

INTRODUCTION

In Physics 112 we will be making heavy use of vectors, so we will start with a review of vectors. The important vector quantities we will be discussing are electric forces, electric fields, electric currents, magnetic forces, magnetic fields, and the velocity of propagation of electromagnetic waves.

We will also be making heavy use of the concepts of mechanical energy, kinetic energy, potential energy, and work – both conservative and nonconservative – so we will review these, emphasizing gravitational forces.

Topic 1A. Vectors

PHYSICAL QUANTITIES

Physics deals over and over with physical quantities – quantities such as mass, temperature, force, acceleration, moment of inertia. Sometimes you are given their values in a problem. Sometimes you have to measure them, as in a laboratory. Sometimes you calculate them from other quantities. Sometimes you predict what their values will be at some time in the future, say 15.0 seconds from now, or when a object rolling down a hill reaches the bottom of the hill.

SCALAR PHYSICAL QUANTITIES

Some physical quantities are just a number, without units. An example of such a physical quantity is the coefficient of kinetic friction. It has a value such as 0.55 or 0.13, with no units of any kind. That's because it is really a ratio of two forces, so the units cancel out. These quantities are referred to as ***scalars***.

Other physical quantities are specified by a numerical value and a unit. Together, the numerical value and the unit are referred to as the magnitude of the quantity. For example, an object might have a mass of 5.0 kilograms. "5.0" is the numerical value of this mass, and "kilograms" refers to the unit. With different units the numerical value is different, but the magnitude is the same. This mass, for example, could also be said to have a mass of 0.0050 megagrams, or 5000 grams. These quantities are also referred to as scalars.

VECTOR PHYSICAL QUANTITIES

Some physical quantities have a magnitude but also require the specification of a direction in space. An example is the force acting on an object. It might be "4.7 newtons south" or "2500 newtons in an upward direction." Or the velocity of an automobile might be "55 miles per hour east" or "60 miles per hour west." Physical quantities that require a direction in order to be completely specified as referred to as ***vectors***.

In summary, then, a scalar is a quantity that is completely specified by a magnitude (whether or not it has units). A vector is a quantity that is completely specified by a magnitude and a direction. The magnitude of a vector is itself a scalar.

Question 1. Here are some physical quantities that were introduced and used in Physics 111. Indicate for each if it is (1) a scalar or (2) a vector.

A. Mass	B. Kinetic energy	C. Force
D. Potential energy	E. Velocity	F. Speed
G. Acceleration	H. Linear momentum	I. Displacement
J. Distance traveled	K. Heat capacity	L. Entropy

SYMBOLS USED TO REPRESENT VECTORS

We'll always denote a vector by a symbol with an arrow over it, like $\vec{F}$ or $\vec{v}$. The arrow reminds us that it is a vector, and not a scalar quantity. Two vectors are equal if and only if they have the *same magnitude* and the *same direction*.

The ***magnitude*** of the vector $\vec{A}$ will be denoted by $|\vec{A}|$ or often just A. It is a scalar.

REPRESENTING VECTORS GEOMETRICALLY (BY ARROWS)

Vectors can be nicely represented as arrows pointing in the correct direction, the lengths of the vectors being proportional to their magnitudes. This is referred to as the "geometrical representation of the vectors."

Example 1. A vector $\vec{A}$ pointing to the right might be represented by:

A vector $\vec{B}$ pointing to the left, with twice the magnitude of $\vec{A}$, might be represented by:

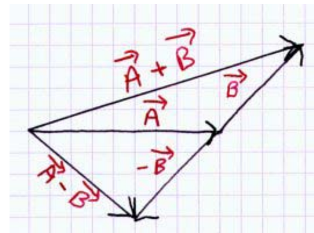
It is often necessary to add or subtract vectors. Here's how to add two vectors geometrically: represent each vector by an arrow of the appropriate length and direction, and add them by placing them head to tail (in any order), the sum being a vector from the tail of the first vector to the head of the last vector. To do this, move the vector arrows around without changing the direction in which they point. The vector sum of two vectors $\vec{A}$ and $\vec{B}$ is denoted $\vec{A}+\vec{B}$. An example in which vectors are added is the determination of the total force acting on a object, which is the sum of the individual forces (which we might denote $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, etc.) acting on the object: $\vec{F}_{\text{total}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$.

The difference of two vectors is defined as: $\vec{A} - \vec{B} \equiv \vec{A} + (-\vec{B})$, where $-\vec{B}$ is the vector equal in magnitude to, but opposite in direction to, $\vec{B}$. An example of the use of subtraction is when the velocity of object 2 is determined relative to an object 1: it is just $\vec{v}_1 - \vec{v}_2$.

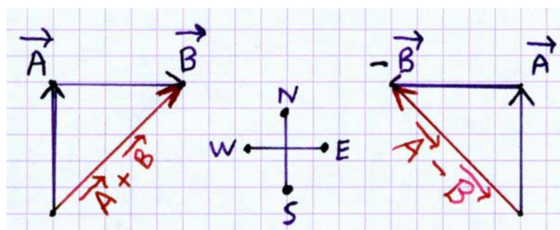
Note: Vectors can be added to (or subtracted from) each other only if they represent the same physical quantity and thus have the same units. You can add a force to a force but not a force to a velocity.

Example 2. The sum of ⟵―― and ⟶ is ⟵.

Example 3. The diagram to the right shows two vectors $\vec{A}$ and $\vec{B}$ and their sums and differences. Note that with $\vec{A}$ along the positive x axis and $\vec{B}$ in the first quadrant, the sum $\vec{A}+\vec{B}$ is in the first quadrant while the difference $\vec{A}-\vec{B}$ is in the fourth quadrant.



Example 4. Consider a vector $\vec{A}$ of magnitude 5.0 cm directed north and a vector $\vec{B}$ of magnitude 5.0 cm directed east. (a) Form the sum $\vec{A}+\vec{B}$ geometrically and estimate its magnitude and direction. (b) Find $\vec{A}-\vec{B}$ geometrically and estimate its magnitude and direction.



The sum $\vec{A}+\vec{B}$ has a magnitude of approximately 7 cm and a direction that is exactly towards the northeast. The difference $\vec{A}-\vec{B}$ also has a magnitude of approximately 7 cm and its direction is towards the northwest.

Question 2. Which of the vectors labeled C, D, E, or F is most nearly in the same direction as the vector sum $\vec{A}+\vec{B}$ of the vectors $\vec{A}$ and $\vec{B}$?

$\vec{A}$ ↑ $\vec{B}$ → (1) ↗ (2) ↘ (3) ↙ (4) ↖

Question 3. Which of the vectors labeled C, D, E, or F is most nearly in the same direction as the vector sum $\vec{B}+\vec{A}$ of the vectors $\vec{A}$ and $\vec{B}$?

$\vec{A}$ ↑ $\vec{B}$ → (1) ↗ (2) ↘ (3) ↙ (4) ↖

Question 4. Which of the vectors labeled C, D, E, or F is most nearly in the same direction as the vector difference $\vec{A}-\vec{B}$?

$\vec{A}$ ↑ $\vec{B}$ → (1) ↗ (2) ↘ (3) ↙ (4) ↖

Question 5. Which of the vectors labeled C, D, E, or F is most nearly in the same direction as the vector difference $\vec{B}-\vec{A}$ of the vectors $\vec{A}$ and $\vec{B}$?

$\vec{A}$ ↑ $\vec{B}$ → (1) ↗ (2) ↘ (3) ↙ (4) ↖

SCALAR MULTIPLICATION OF VECTORS

The multiplication of a vector $\vec{A}$ by a scalar c is a new vector $c\,\vec{A}$ whose magnitude is cA (c times the magnitude of $\vec{A}$) and whose direction is either the same as that of $\vec{A}$ (if $c > 0$) or opposite to that of $\vec{A}$ (if $c < 0$). The scalar c can be a pure number or a number with units.

Example 5. This shows the doubling of a vector: $2\,(\longrightarrow) = \longrightarrow\!\!\!\longrightarrow$.

Consider a vector $\vec{A}$ of magnitude 3.0 cm directed along the positive x axis and a vector $\vec{B}$ of magnitude 4.0 cm directed along the negative x axis.

Question 6. **Form the sum $\vec{A} + \vec{B}$ geometrically and estimate its magnitude and direction.**

Magnitude: (1) 1.0 cm (2) 2.0 cm (3) 4.5 cm (4) 5.0 cm (5) 7.0 cm.

Direction: (1) Towards positive x (2) Towards negative x

Question 7. **Find $\vec{A} - \vec{B}$ geometrically and estimate its magnitude and direction.**

Magnitude: (1) 1.0 cm (2) 2.0 cm (3) 4.5 cm (4) 5.0 cm (5) 7.0 cm.

Direction: (1) Towards positive x (2) Towards negative x

Consider a vector $\vec{A}$ of magnitude 3.0 cm directed south and a vector $\vec{B}$ of magnitude 4.0 cm directed west.

Question 8. **Form the sum $\vec{A} + \vec{B}$ geometrically and estimate its magnitude and direction.**

Approximate magnitude:

(1) 1.0 cm (2) 2.0 cm (3) 4.5 cm (4) 5.0 cm (5) 7.0 cm.

Approximate direction:

(1) Northwest (2) Northeast (3) Southeast (4) Southwest

Question 9. **Find $\vec{A} - \vec{B}$ geometrically and estimate its magnitude and direction.**

Approximate magnitude:

(1) 1.0 cm (2) 2.0 cm (3) 4.5 cm (4) 5.0 cm (5) 7.0 cm.

Approximate direction:

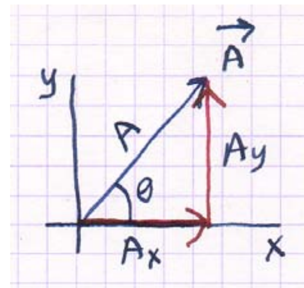
(1) Northwest (2) Northeast (3) Southeast (4) Southwest

REPRESENTING VECTORS ANALYTICALLY

For most purposes it is more convenient to represent vectors analytically, using two or three numbers, than geometrically, with arrows. Here's how it's done.

Two-dimensional vectors

Imagine that you are dealing with two-dimensional vectors, say vectors in the xy plane. Instead of representing the vector by an arrow of the correct length pointing in the appropriate direction, imagine representing it as the sum of two vectors, one of which is along the x (or $-x$ axis) and the other along the y (or $-y$) direction, as shown in the diagram on the right.



Now we can represent the vector by two numbers, one corresponding to the length A_x of the vector in the x direction (with a $-$ sign if it is actually along $-x$) and the other corresponding to the length A_y in the y direction (with a $-$ sign if it is actually along $-y$). This is usually expressed compactly by writing: $\vec{A} = (A_x,\ A_y)$, which is referred to as the analytic representation of the vector. Knowing A_x and A_y is equivalent to knowing the actual vector, because no other vector could have the same values of A_x and A_y. A_x and A_y are referred to as the components of the vector $\vec{A}$ along the x and y directions.

By the Pythagorean theorem, we can see that the magnitude (length) of vector $\vec{A}$, expressed in terms of its components, is just

$$\Longrightarrow \qquad \boldsymbol{A = \sqrt{(A_x)^2 + (A_y)^2}.}$$

A 2-dimensional vector can also be represented using polar coordinates, in terms of its magnitude A and the angle θ (measured counterclockwise) between the x-axis and the direction of the vector:

$$\Longrightarrow \qquad \boldsymbol{A = \sqrt{(A_x)^2 + (A_y)^2}} \ \text{ and } \ \boldsymbol{\tan\theta = A_y/A_x.}$$

$$\Longrightarrow \qquad \boldsymbol{A_x = A\cos\theta} \text{ and } \boldsymbol{A_y = A\sin\theta.}$$

Components make it easy to add or subtract vectors or multiply them by scalars: you just add or subtract the components or multiply them by the appropriate scalar. The components of the sum or difference of two vectors are just the sums or differences of the components of those two vectors:

$$\vec{A} \pm \vec{B} = (A_x \pm B_x,\ A_y \pm B_y).$$

More generally, if c and d are scalars,

$$\Longrightarrow \qquad \boldsymbol{c\vec{A} \pm d\vec{B} = (cA_x \pm dB_x,\ cA_y \pm dB_y).}$$

Three-dimensional vectors

In three dimensions we do exactly the same thing, representing a vector as the sum of three vectors along the $x, y,$ and z axes. We then have three components A_x, A_y, and A_z and we write: $\vec{A} = (A_x,\ A_y,\ A_z)$.

In three dimensions the magnitude of $\vec{A}$ is $A = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$ and the components of the sum or difference of two vectors are just the sums or differences of the components of those two vectors:

$$c\vec{A} \pm d\vec{B} = (cA_x \pm dB_x,\ cA_y \pm dB_y, cA_z \pm dB_z).$$

Example 6. **Find the components of $2\vec{A} - 3\vec{B}$ in terms of the components of $\vec{A}$ and $\vec{B}$.**

$$2\vec{A} - 3\vec{B} = (2A_x - 3B_x, 2A_y - 3B_y,\ 2A_z - 3B_z).$$

Unit vectors

In terms of its components A_x, A_y, and A_z, the vector $\vec{A}$ can be represented either by its three components in parentheses or in terms of unit vectors $\widehat{\mathbf{x}}$, $\widehat{\mathbf{y}}$, $\widehat{\mathbf{z}}$:

$\Longrightarrow$ $$\vec{A} = (A_x, A_y, A_z) \quad \text{or} \quad \vec{A} = A_x\,\widehat{\mathbf{x}} + A_y\,\widehat{\mathbf{y}} + A_z\,\widehat{\mathbf{z}}$$

The unit vectors $\widehat{\mathbf{x}}$, $\widehat{\mathbf{y}}$, $\widehat{\mathbf{z}}$ (denoted $\widehat{\mathbf{i}}$, $\widehat{\mathbf{j}}$, $\widehat{\mathbf{k}}$ in some textbooks) are vectors of length 1 (just "1" – not 1 meter or 1 inch or anything else) pointed along the positive $x, y,$ and z axes, respectively. Unless you are specifically told which way to represent a vector, you may use either representation, whichever you prefer.

When we write a vector in unit vector notation, we are making explicit the fact that there are three vectors along the coordinate axes that will add up to this vector: $\vec{A}$ is the sum of $A_x\,\widehat{\mathbf{x}}$ (a vector of length A_x along $\widehat{\mathbf{x}}$) plus $A_y\,\widehat{\mathbf{y}}$ (a vector of length A_y along $\widehat{\mathbf{y}}$) plus $A_z\,\widehat{\mathbf{z}}$ (a vector of length A_z along $\widehat{\mathrm{z}}$).

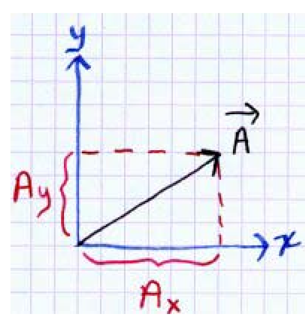
The component of a vector along a specified direction is the length of the projection of the vector along that direction. The projection of the vector along an axis can be thought of as the shadow of the vector on the axis from a distant light source whose light rays are perpendicular to the axis. In some countries, the term "vector projections" is used instead of "vector components."

The component of a vector can be negative, zero, or positive. It is positive if it is a projection onto a positive axis and negative if it is a projection onto a negative axis.

In the Cartesian coordinate system the components are:

$A_x =$ component along x direction
$A_y =$ component along y direction
$A_z =$ component along z direction

The diagram on the right show the x and y components of a vector in the xy plane. We will use only right-hand coordinate systems, for which the positive z axis would be directed up from the plane of the page.



In terms of the magnitude A of a vector $\vec{A}$ and the angle θ it makes with the positive x axis, the x and y components are given by $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Example 7. **Find the components of a vector $\vec{A}$ of length 5.0 cm directed at an angle of 60° (measured, as always, counterclockwise from the positive x axis).**

The x component is: $A_x = A\cos\theta = (5.0\text{ cm})\cos 60° = (5.0\text{ cm})(0.500) = 2.5\text{ cm}$.

The y component is: $A_y = A\sin\theta = (5.0\text{ cm})\sin 60° = (5.0\text{ cm})(0.866) = 4.3\text{ cm}$

Example 8. **Find the magnitude and angle of the vector $\vec{B} = -(3.0\text{ m})\,\hat{\mathbf{x}} + (4.0\text{ m})\,\hat{\mathbf{y}}$.**

The magnitude is: $B = |\vec{B}| = \sqrt{(-3.0\text{ m})^2 + (4.0\text{ m})^2} = \sqrt{(9.0\text{ m}^2) + (16.0\text{ m}^2)} = 5.0\text{ m}$.

Its angle is: $\theta = \arctan(B_y/B_x) = \arctan(4.0/(-3.0)) = \arctan(-1.33) = -57°$ or $123°$.

Example 9. **Determine the sign of the x and y components of vectors in the first, second, third, and fourth quadrants (the quadrants going counterclockwise from the positive x axis towards the positive y axis).**

Component	1st quadrant	2nd quadrant	3rd quadrant	4th quadrant
x component	$+$	$-$	$-$	$+$
y component	$+$	$+$	$-$	$-$

Example 10. **On earth a "local" right-handed coordinate system is normally defined by taking the positive x, y, and z directions to be along east, north, and up from the surface of the earth at that location (so the negative x, y, and z directions are along west, south, and down). Consider a vector with components that are 5.0 m, -6.0 m, and 3.0 m along x, y, and z. Express this vector in terms of its components and also determine its magnitude.**

This vector is $(5.0\text{ m}, -6.0\text{ m}, 3.0\text{ m})$.

Its magnitude is $\sqrt{(5.0\text{ m})^2 + (-6.0\text{ m})^2 + (3.0\text{ m})^2} = \sqrt{70.0\text{ m}^2} = 8.4\text{ m}$.

The sum $\vec{A} + \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ has Cartesian components which are the sums of the respective Cartesian components of $\vec{A}$ and $\vec{B}$:

$$\vec{A} + \vec{B} = (A_x + B_x,\, A_y + B_y,\, A_z + B_z).$$

More generally:

$$k_1\vec{A} + k_2\vec{B} = (k_1A_x + k_2B_x,\, k_1A_y + k_2B_y,\, k_1A_z + k_2B_z),$$

where k_1 and k_2 are constants (positive, negative, or zero). For example,

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Question 10. **Find the sum of these three force vectors:**
$\vec{A} = (-2.0\text{ N}, 0, 4.0\text{ N})$, $\vec{B} = (3.5\text{ N}, 7.2\text{ N}, -2.5\text{ N})$, $\vec{C} = (4.3\text{ N}, -3.5\text{ N}, 0)$.

Question 11. **Suppose that** $\vec{A} = (-2.0\text{ N}, 0, 4.0\text{ N})$, $\vec{B} = (3.5\text{ N}, 7.2\text{ N}, -2.5\text{ N})$, **and** $\vec{C} = (4.3\text{ N}, -3.5\text{ N}, 0)$.

★ **The magnitude of the vector $\vec{B}$ is: (1) 8.2 (2) 8.4 (3) 12.1 (4) 13.2**

★ **Determine the value of $2\vec{A} - \vec{B} + \vec{C}$, expressing it like the three vectors above.**

Question 12. **Suppose** $\vec{v}_1 = (-2.0\text{ m/s}, 1.0\text{ m/s}, 3.0\text{ m/s})$ **and** $\vec{v}_2 = (2.1\text{ m/s}, 0, -2.5\text{ m/s})$.

★ **Determine the value of** $\vec{v}_2 - \vec{v}_1$:

★ **Determine the magnitudes of the following vectors:**

$|\vec{v}_1| =$

$|\vec{v}_2| =$

$|\vec{v}_2 - \vec{v}_1| =$

Topic 1B. Newton's Laws of Motion

NEWTON'S FIRST LAW OF MOTION

Newton's first law of motion states that in an inertial reference frame an object at rest will remain at rest, and an object in motion will continue in motion with a constant velocity, unless there is a nonzero net external force acting on the object.

NEWTON'S SECOND LAW OF MOTION

Newton's second law of motion states that the acceleration of an object is directly proportional to the net (total) force acting on it and inversely proportional to its mass m. The mathematical statement of this law is $\sum \vec{F}_i = m\vec{a}$ or $\vec{F}_{\text{net}} = m\vec{a}$.

NEWTON'S THIRD LAW OF MOTION

Newton's third law of motion states that if two objects 1 and 2 interact, the force on object 1 exerted by object 2 is equal in magnitude but opposite in direction to the force on object 2 exerted by object 1. In mathematical terms,

$\vec{F}_{12} = -\vec{F}_{21}$, where $\vec{F}_{ij}$ is the force on object i exerted by object j.

Note the notation that we are using here: $\vec{F}_{12}$ means "the force on object 1 exerted by object 2." Always try to read the notation this way.

object 1 $\circ \longrightarrow \vec{F}_{12}$ $\qquad\qquad$ $\vec{F}_{21} \longleftarrow \blacksquare$ object 2

Newton's Third Law applies not only to the total force between two objects, but to each and every interaction between them. For example, if the two objects have both mass and non-zero electric charge, then Newton's third law applies separately to both the gravitational and electrical forces between them.

The pair of forces $\vec{F}_{12}$ and $\vec{F}_{21}$ is said to form an "action-reaction" pair. Note three important things about this pair of forces:

- The forces are equal in magnitude and opposite in direction. Always!
- They act on different objects. This is very important to remember! One of the forces acts on object 1 and the other acts on object 2. The two forces can't both appear on the same free-body force diagram unless the system of both objects is being considered.
- The fundamental interaction responsible for $\vec{F}_{12}$ must be the same as the fundamental interaction for $\vec{F}_{21}$. They are both gravitational, or both electromagnetic, for example. It isn't possible for one to be gravitational and the other to be electromagnetic.

Topic 1C. The Gravitational Force

THE GRAVITATIONAL FORCE

The gravitational force: **According to Newton's law of gravitation, every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. Mathematically, the magnitude of this force is**

$$F_g = \frac{G\, m_1\, m_2}{r^2}$$

where $G = 6.672 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ is the universal gravitational constant.

$$m \circ\!\longrightarrow \vec{F}_g \qquad\qquad \vec{F}_g \longleftarrow\!\bigcirc\ M$$

The gravitational forces exerted by two objects on one another.
The gravitational force on each object is directed towards the other mass.
The gravitational forces have the same magnitude but opposite directions.
The object with the smaller mass will have the greater acceleration.

Example 11. **Estimate the magnitude of the gravitational force between the Earth and the Moon.**

Using the masses of the Earth (5.98×10^{24} kg) and of the Moon (7.36×10^{22} kg) and the mean distance (3.84×10^8 m) between the two, the magnitude of the gravitational force is

$$\frac{G\, m_{\text{earth}}\, m_{\text{moon}}}{r^2} = \frac{(6.67\times10^{-11}\ \mathrm{N{\cdot}m^2/kg^2})(5.98\times10^{24}\ \mathrm{kg})(7.36\times10^{22}\ \mathrm{kg})}{(3.84\times10^{8}\ \mathrm{m})^2} \approx 2\times10^{20}\ \mathrm{N}.$$

The gravitational force is always attractive between any two pieces of matter. Presumably it is also attractive between antimatter and antimatter. But what about between matter and antimatter? There is no good direct experimental information about this, but theoretical physicists firmly believe that antimatter and matter will also gravitationally attract each other. So there would be nothing along the lines of anti-gravity. Nevertheless, there are some indications from astronomy that some type of repulsion might exist in the universe, but this is not well-understood.

Outside of a spherically symmetric object of total mass M, the gravitational force exerted by that object on another is the same as that of a particle of mass M located at the center of the spherically symmetric object.

At a distance $r < R$ from the center of the spherically symmetric object, the gravitational force is that of a single object, located at the center, whose mass equals the mass of the part of the spherically symmetric object that is within a distance r of its center. Large astronomical objects (stars, Sun, planets, Moon) are very nearly spherically symmetric

Example 12. **Estimate the magnitude of the gravitational force between the Moon and the Sun Compare this with the magnitude of the gravitational force between the Moon and the Earth.**

$$F_{\text{sun-moon}} = \frac{G\, m_{\text{sun}}\, m_{\text{moon}}}{r^2} = \frac{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(7.36\times10^{22}\ \text{kg})}{(1.5\times10^{11}\ \text{m})^2} \approx 4\times10^{20}\ \text{N}.$$

This is about twice the magnitude of the gravitational force between the Moon and the Earth. In a sense, the Moon is revolving around the Sun more than it is revolving around the Earth. We can say that the Moon is revolving around the Sun in an orbit perturbed by the Earth.

The gravitational field $\vec{g}$ at a point in space is defined by $\vec{g} = \vec{F}_g/m$, where $\vec{F}_g$ is the total gravitational force at that point, due to all objects in the universe, acting on a mass m placed at that point.

The gravitational field $\vec{g}$ at a point in space is defined as the ratio of the gravitational force on a mass m located at that point divided by m. This definition gives the same result for $\vec{g}$ no matter what mass is used. Near the surface of the earth, where the gravitational force on an object is mainly the gravitational force due to the earth, this field has the magnitude GM/R_e^2, where M is the mass of the earth and R_e is the radius of the earth; the value of this constant is 9.8 N/kg. The concept of a field is a very important one, and will be used even more in dealing with electric and magnetic forces, where the electric and magnetic fields will be introduced and used.

Example 13. **Determine an expression for the magnitude of the gravitational field at the surface of the Earth, assuming that only the gravitational force of the earth is significant. Evaluate it.**

Consider an object of mass m located at the surface of the Earth, whose mass is M and whose radius is R. The magnitude of the gravitational force is

$$g = \frac{F_g}{m} = \frac{GMm/R^2}{m} = \frac{GM}{R^2} = \frac{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(6.4\times10^{3}\ \text{m})^2} \approx 9.8\ \text{N/kg}.$$

Question 13. **Estimate the magnitude of the gravitational *force* between the Earth and the Sun and compare it, using a ratio, with the magnitude of the gravitational force between the Earth and the Moon. Remember the table of astronomical data on the first page of this topic.**

The magnitude is (to two significant figures):

The ratio is about: (1) 25 (2) 75 (3) 125 (4) 175 (5) 225

<u>Question 14.</u> Determine the contributions of the Moon and the Sun to the total gravitational *<u>field</u>* at the surface of the Earth; calculate the actual N/kg. How important are these compared to the contribution of the Earth? Ignore the directions of these forces.

g due to the Earth is about: 9.8 N/kg [See Example 3 above, or recalculate it yourself.]

g due to the Moon is about:

g due to the Sun is about:

Questions 15 through 17 involve ratios. Don't calculate the actual forces, just write down the expressions for the forces and form ratios. The ability to form ratios is very important in physics and will often save you from doing a lot of calculations and possibly making errors..

<u>Question 15.</u> A 5-kg lead sphere is hanging 1.4 m from a 500-kg lead sphere. How does the gravitational force exerted by the 5-kg sphere on the 500-kg sphere compare with the magnitude of the gravitational force exerted by the 500-kg sphere on the 5-kg sphere? The forced exerted by the 5-kg sphere on the 500-kg sphere is:

(1) 100 times larger
(2) 10 times larger
(3) exactly the same
(4) 1/10 as large
(5) 1/100 as large

<u>Question 16.</u> Suppose the distance between the spheres in #3 is increased to 4.2 m. What happens to the magnitude of the gravitational force exerted by the 5-kg sphere on the 500-kg sphere?

(1) It becomes 9 times larger than it was before.
(2) It becomes 3 times larger than it was before.
(3) It is exactly the same as it was before.
(4) It becomes one-third as large as it was before.
(5) It becomes one-ninth as large as it was before.

<u>Question 17.</u> Suppose the distance between the two spheres in #3 remains the same (1.4 m), but somehow the mass of the smaller sphere is changed to 20 kg. What would happen to the magnitude of the gravitational force exerted on the 500-kg sphere?

(1) It becomes four times larger than it was before.
(2) It becomes twice as large as it was before
(3) It would be exactly the same as it was before.
(4) It becomes one-half as large as it was before.
(5) It becomes one-fourth as large as it was before.

Consider these three situations:

Case X. A 3-kg lead sphere is 2.0 m away from a 5-kg lead sphere.
Case Y. A 3-kg lead sphere is 4.0 m away from a 10-kg lead sphere.
Case Z. A 3-kg lead sphere is 6.0 m away from a 15-kg lead sphere.

It is possible to calculate the gravitational forces in each case from Newton's law of gravitation, but do not do that. Instead, uses ratios to answer these questions.

Question 18. **Suppose the magnitude of the gravitational force on the 3-kg sphere in situation X is denoted by F. Determine the relative magnitudes of the gravitational force on the 3-kg sphere due to the other sphere in the other cases.**

Case Y:	(1) $F/4$	(2) $F/2$	(3) F	(4) $2F$	(5) $4F$
Case Z:	(1) $F/9$	(2) $F/3$	(3) F	(4) $3F$	(5) $9F$

Question 19. **What can you say about the magnitude of the gravitational force on the 3 kg sphere in these three situations? The magnitude of the gravitational force on the 3 kg sphere is:**

(1) largest in case X (3 and 5 kg spheres)
(2) largest in case Y (3 and 10 kg spheres)
(3) largest in case Z (3 and 15 kg spheres)
(4) equal in cases X and Y, but larger than in case Z
(5) equal in cases Y and Z, but larger than in case X
(6) equal in all three cases

Topic 1D. Energy

REVIEW OF WORK AND ENERGY

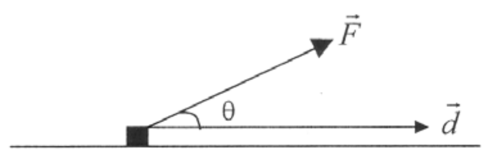
Important Definitions:

1. ***Work (W)*** is a measure of the energy added to an object through the action of a force;
2. A ***conservative force*** is a force whose action does not alter an object's total energy;
3. ***Potential Energy (PE)*** is the energy associated with the ***position*** of an object;
4. ***Kinetic Energy (KE)*** is the energy associated with the ***motion*** of an object;
5. ***Total Mechanical Energy (TE)*** of an object is the sum of its kinetic and potential energies;

Defining Relationships:

Work: $$\boldsymbol{W = Fd\cos\theta}$$

In the equation for work (W), F is the magnitude of the force which is acting on some object, d is the magnitude of the displacement of the object while it is being acted on by that force, and θ is the angle between the direction of the force, and the direction of the displacement. The magnitude of displacement d is equal to the distance the object moves, ***if*** it moves in a straight line. ***This equation for work assumes that the force is constant, and it may not be used in situations where the force changes in magnitude or direction while the object is moving.***



This diagram shows an object being pulled along a surface by a force $\vec{F}$. For example, this might be a box pulled by a force applied to a rope held up at an angle θ relative to the horizontal.

Forces are classified as conservative or non-conservative in the following way:

1. A ***conservative force*** is one for which the work done in moving an object from one position to another does not depend on the path used. Gravitational, electrical, and magnetic forces are conservative, as is the elastic force of an ideal spring..

2. A ***non-conservative force*** is one which is not conservative. Examples are friction and other dissipative forces, and push or pull due to direct contact with another object or a person

The total or net work done on an object is the sum of all the works done on it by individual forces. This can be divided into two parts, the work W_{cons} done by conservative forces and the work $W_{\text{non-cons}}$ done by non-conservative forces.

Potential Energy

The potential energy function PE associated with a conservative force is defined (to within a constant) by the formula $\Delta PE = -W_{\text{cons}}$. Simple expressions can be derived for many important conservative forces:

- $PE = mgy$ for an object of mass m in a constant gravitational field of magnitude g in the $-y$ direction; this is an adequate approximation for the gravitational potential energy near the earth's surface. (The constant is chosen so $PE = 0$ at $y = 0$.)
- $PE = \frac{1}{2}kx^2$ for a spring with spring constant k whose end is displaced from its equilibrium position by x. (The constant is chosen so $PE = 0$ at $x = 0$.)
- $PE = -GmM/r$ for the gravitational potential energy of two objects of masses m and M separated by a distance r. (The constant is chosen so $PE = 0$ at $r = \infty$.)
- $PE = kq_1q_2/r$ for the electrical potential energy of two objects of electric charge q_1 and q_2 separated by a distance r, as will be discussed in Topic 2. (The constant is chosen so $PE = 0$ at $r = \infty$.)

Kinetic Energy: $\boldsymbol{KE = \frac{1}{2}mv^2}$

In this equation for kinetic energy (KE), m is the mass of the object, and v is the speed of the object; kinetic energy is a scalar even though it depends on a vector (the velocity).

Total Mechanical Energy: $\boldsymbol{TE = KE + PE}$

Everything above consists of ***definitions,*** which have no physical content. The fundamental physical principles themselves can be summarized in only two equations:

1. **The work-energy theorem:** $\boldsymbol{W_{\text{net}} = \Delta KE}$
2. **Conservation of Energy:** $\boldsymbol{\Delta TE = W_{\textit{non-cons}}}$

Here, ΔKE is the change in kinetic energy: $\Delta KE = (KE)_f - (KE)_i$.
ΔTE is the change in total energy: $\Delta TE = (TE)_f - (TE)_i = \Delta KE + \Delta PE$.
$W_{\text{net}} = W_{\textit{cons}} + W_{\textit{non-cons}} = -\Delta PE + W_{\textit{non-cons}}$

The "Work-Energy" theorem says that the change in the kinetic energy of an object is equal to the net work done on that object. So, for instance, if positive work is done, the object will speed up. (Note: The change in KE is actually equal to the net work, which can be calculated either as the sum of the works done by all the different forces acting on the object, or as the work don by the net (total) force acting on the object.)

The Conservation of Energy equation says that any change in total energy of an object is equal to work done only by nonconservative forces on that object. This implies that if only conservative forces are acting on the object, its total mechanical energy $(KE + PE)$ does not change.