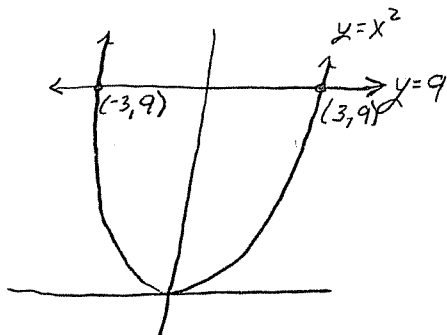


Quiz #5a: Sections 5.6 & 5.7

Show all work in a neat and logical manner in order to get full credit.
You may use your calculator; however all answers should be exact!

- 15 pts. 1. Find the centroid of the region bounded by the curves $y = x^2$ and $y = 9$. If you use symmetry, explain how and why you are able to use it.



$\bar{x} = 0$ by symmetry. $x = 0$ is the axis of symmetry of this parabola. Also, if you folded this region in half vertically and have the same area on both sides.

You can't use symmetry to find \bar{y} because the area towards the top of the region is larger than the area towards the bottom of the region.

$$\bar{y} = \frac{\frac{1}{2} \int_{-3}^3 (9^2 - (x^2)^2) dx}{\int_{-3}^3 9 - x^2 dx} = \frac{\frac{1}{2} \int_{-3}^3 81 - x^4 dx}{\int_{-3}^3 9 - x^2 dx} = \frac{\frac{1}{2} \left(81x - \frac{x^5}{5} \right) \Big|_{-3}^3}{9x - \frac{x^3}{3} \Big|_{-3}^3} = \frac{\frac{1}{2} \left(\frac{972}{5} - \left(-\frac{972}{5} \right) \right)}{18 - (-18)} = 5.4$$

$$(\bar{x}, \bar{y}) = \frac{(0, 5.4)}{(0, \frac{27}{5})}$$

Don't forget the back! \Rightarrow

15 pts. 2. Given the probability distribution function

$$f(x) = \begin{cases} \frac{x^3}{64} & \text{if } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(1 \leq X \leq 3)$

$$\int_1^3 \frac{x^3}{64} dx = \left. \frac{x^4}{256} \right|_1^3 = \frac{81}{256} - \frac{1}{256} = \frac{80}{256} = \frac{5}{16} = .3125$$

(b) Find $E(X)$

$$\int_0^4 x \left(\frac{x^3}{64} \right) dx = \int_0^4 \frac{x^4}{64} dx = \left. \frac{x^5}{320} \right|_0^4 = \frac{1024}{320} = 3.2$$

Points earned: _____ out of a possible 30 points