

Quiz #3b: Sections 5.3 & 5.4

Show all work in a neat and logical manner in order to get full credit.

10 pts.

1. Set-up the integral to find the length of the curve  $x = \frac{2}{3}t^3$ ,  $y = 3t^2$  on  $0 \leq t \leq 6$ . Simplify your answer, so that the integration will be easy. You do not have to evaluate the integral.

$$L = \int_a^b \sqrt{(x')^2 + (y')^2} dt$$

$$x = \frac{2}{3}t^3$$

$$y = 3t^2$$

$$L = \int_0^6 \sqrt{(2t^2)^2 + (6t)^2} dt$$

$$x' = 2t^2$$

$$y' = 6t$$

$$L = \int_0^6 \sqrt{4t^4 + 36t^2} dt$$

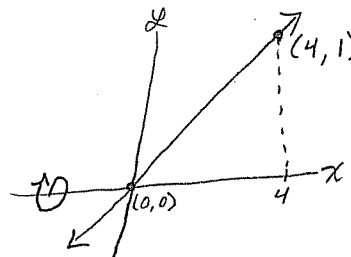
$$L = \int_0^6 \sqrt{4t^2} \sqrt{t^2 + 9} dt$$

$$L = \int_0^6 2t \sqrt{t^2 + 9} dt$$

10 pts.

2. Find the lateral (side) surface area of the cone generated by revolving the line segment  $y = \frac{1}{4}x$ ,  $0 \leq x \leq 4$ , about the x-axis. (Include a graph and round your answer to the nearest tenth.)

$$\begin{aligned}
 A &= 2\pi \int_a^b g(x) \sqrt{[f'(x)]^2 + [g'(x)]^2} dx \\
 A &= 2\pi \int_0^4 \frac{1}{4}x \sqrt{1 + \left(\frac{1}{4}\right)^2} dx \\
 &= \frac{4\pi}{2} \sqrt{\frac{17}{16}} \int_0^4 x dx \\
 &= \frac{\pi}{8} \sqrt{17} \left(\frac{x^2}{2}\right)_0^4 \\
 &= \pi \sqrt{17} \\
 &\approx 13
 \end{aligned}$$



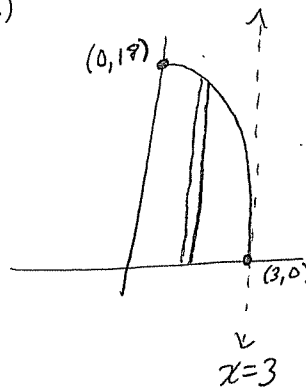
10 pts.

3. Use the shell method to find the volume of the solid generated when the region bounded by the given curves is revolved about the line  $x = 3$ .

$$y = 18 - 2x^2 \quad (x \geq 0), \quad x = 0, \quad y = 0$$

(Include a graph and find an exact answer using  $\pi$ , if needed.)

$$\begin{aligned}
 \text{Shell Method: } & 2\pi \int_a^b r \cdot h \, dr \\
 V &= 2\pi \int_0^3 (3-x)(18-2x^2) dx \\
 &= 2\pi \int_0^3 54 - 18x - 6x^2 + 2x^3 dx \\
 &= 2\pi \left( 54x - 9x^2 - 2x^3 + \frac{x^4}{2} \right)_0^3 \\
 &= 135\pi
 \end{aligned}$$



Points earned: \_\_\_\_\_ out of a possible 30 points