Math 166 - Section A
February 1, 2008

Name: Key - Green

Quiz #3a: Sections 5.3 & 5.4
Show all work in a neat and logical manner in order to get full credit.

10 pts. 1. Set-up the integral to find the length of the curve $x = \frac{2}{3}t^3$, $y = 12t^2$ on $0 \le t \le 3$. Simplify your answer, so that the integration will be easy. You do not have to evaluate the integral.

$x = \frac{2}{3}t^3 \qquad y = 12t^2$

$x' = 2t^2 \qquad y' = 24t$

$$L = \int_a^b \sqrt{(x')^2 + (y')^2}\, dt$$

$$L = \int_0^3 \sqrt{(2t^2)^2 + (24t)^2}\, dt$$

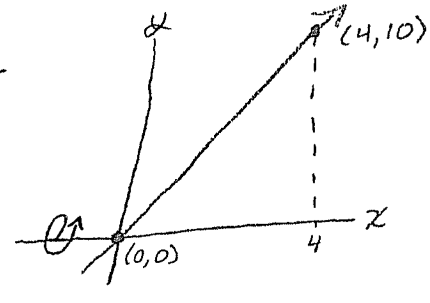
$$= \int_0^3 \sqrt{4t^4 + 576t^2}\, dt$$

$$= \int_0^3 \sqrt{4t^2}\sqrt{t^2 + 144}\, dt$$

$$= \int_0^3 2t\sqrt{t^2 + 144}\, dt$$

Don't forget the back! ⇒

10 pts. 2. Find the lateral (side) surface area of the cone generated by revolving the line segment $y = \frac{5}{2}x$, $0 \le x \le 4$, about the x-axis. (Include a graph and round your answer to the nearest tenth.)



$$A = 2\pi \int_a^b g(t)\sqrt{[f'(t)]^2 + [g'(t)]^2}\,dt$$

$$= 2\pi \int_0^4 \frac{5}{2}x \sqrt{1 + \left[\frac{5}{2}\right]^2}\,dx$$

$$= 5\pi\sqrt{\frac{29}{4}} \int_0^4 x\,dx$$

$$= \frac{5}{2}\pi\sqrt{29}\left(\frac{x^2}{2}\right)\Big|_0^4$$

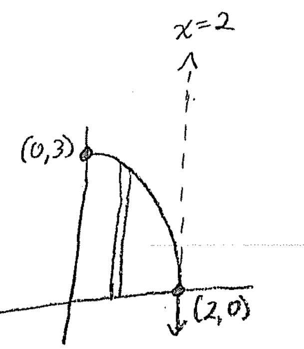
$$= 20\sqrt{29}\,\pi$$

$$\approx 338.5$$

10 pts. 3. Use the shell method to find the volume of the solid generated when the region bounded by the given curves is revolved about the line $x = 2$.

$$y = 3 - \frac{3}{4}x^2 \quad (x \ge 0), \quad x = 0, \quad y = 0$$

(Include a graph and find an exact answer using π, if needed.)



Shell Method: $V = 2\pi \int_a^b r \cdot h\,dr$

$$= 2\pi \int_0^2 (2-x)\left(3 - \frac{3}{4}x^2\right)dx$$

$$= 2\pi \int_0^2 6 - 3x - \frac{3}{2}x^2 + \frac{3}{4}x^3\,dx$$

$$= 2\pi\left(6x - \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{3}{16}x^4\right)\Big|_0^2$$

$$= 2\pi(12 - 6 - 4 + 3)$$

$$= 10\pi$$

Points earned: ____________ **out of a possible 30 points**