A parametric study on reliability-based TMD design against bridge flutter

Filippo Ubertini¹, Gabriele Comanducci¹ and Simon Laflamme²,³

Abstract
We present a probabilistic methodology for designing tuned mass dampers for flutter suppression in long-span bridges. The procedure is computationally efficient and computes the probability of flutter occurrence based on a modified first-order method of reliability analysis, a reduced-order representation of the structure and a time domain formulation of aeroelastic loads. Results of a parametric investigation show that the proposed methodology is preferable to a deterministic design procedure, which relies on nominal values of mechanical and aerodynamic parameters and does not guarantee the maximum safety. Furthermore, the reliability-based approach can be effectively used in the design of multiple tuned mass damper configurations by enhancing robustness against frequency mistuning and by reducing costs associated with supplemental damping for a given safety performance level.

Keywords
Multimode flutter, Multiple tuned mass damper, Structural reliability, Long-span bridge, Aeroelasticity

¹Department of Civil and Environmental Engineering, University of Perugia, Perugia, Italy
²Department of Civil, Construction, and Environmental Engineering, Iowa State University, Ames, IA 50011, USA
³Department of Electrical and Computer Engineering, Iowa State University, Ames, IA 50011, USA

Corresponding author:
Filippo Ubertini, Department of Civil and Environmental Engineering, University of Perugia, Via G Duranti 93, 06125 Perugia, Italy
Email: filippo.ubertini@unipg.it
Introduction

Multimode flutter is an aeroelastic instability resulting from a dynamic coupling between natural modes caused by wind loads. The importance of the flutter phenomena in bridges became infamous with the collapse of the 1940 Tacoma Narrows Bridge. Since then, the flexibility of long-span bridges increased, owing to progress in materials and construction methods, making aeroelastic instabilities, and in particular multimode coupled flutter, major design concerns.

The critical condition of bridge flutter is usually estimated by adopting the classic representation of aeroelastic forces in the mixed time-frequency domain, based on Scanlan’s aeroelastic derivatives (Simiu and Scanlan 1986; Kiviluoma 1999), and accounting for structural modes that contribute to the critical flutter mode (Ding et al. 2002). Time domain formulations can also be used, such as rational function approximation of aeroelastic derivatives and indicial functions (see Salvatori and Borri (2007) for a background on this topic). These approaches significantly simplify the computation of the critical wind speed in comparison with approaches based on aeroelastic derivatives, a substantial computational advantage in reliability analysis.

Multimode flutter analysis involves several uncertain mechanical and aeroelastic parameters. It follows that structural performance against aeroelastic instability is typically expressed in terms of probability of failure (Ge et al. 2000; Pourzeynali and Datta 2002; Cheng et al. 2005; Seo and Caracoglia 2011; Baldomir et al. 2013). Due to the large number of random variables involved in the definition of the limit state function, simulation methods, such as Monte Carlo simulation procedures, fail at providing practicable means for estimating probabilities of failure. As a result, the probability of failure of long-span bridges is typically estimated using approximate methods such as first order reliability methods (Battaini et al. 1998; Nowak and Collins 2000).

Bridge flutter can be suppressed by altering the aerodynamic/aeroelastic behavior of the deck or by introducing supplemental damping. In particular, tuned-mass dampers (TMDs) offer great promise due to their low-cost, passive nature, and high effectiveness around ±15 % of their tuned frequency (Connor and Laflamme 2014; Krenk and Hogsberg 2014). The performance of TMDs was investigated by Gu et al. (1998), Chen and Kareem (2003) for suppressing bridge flutter, and by Lin et al. (2000), Gu et al. (2001), Chen and Cai (2004) for mitigating buffeting vibrations. Recent studies (Casalotti et al. 2014) also investigated the use of non-linear TMDs, such as hysteretic TMDs, and demonstrated some advantages over linearly viscous TMDs. These advantages included a smaller required mass stroke extension and the ability to mitigate the response of the bridge in the post-critical flutter range. These features might lead to broader implementations of passive control devices.

A particular challenge in designing a TMD system for mitigating aeroelastic instabilities is in the estimation of the critical flutter frequency to enable proper tuning, which is difficult to conduct at
an acceptable precision. A solution is to utilize of multiple TMDs (MTMDs). MTMDs allow multi-frequency tuning for improved control robustness, as well as a better distribution of the added inertia mass onto the structure (Casciati and Giuliano 2009; Carpineto et al. 2010). The performance of MTMDs for bridge flutter control has been previously investigated. Kwon and Park (2004) proposed to use irregular MTMDs for flutter control in long-span bridges and presented a design procedure accounting for uncertainties in aeroelastic and mechanical parameters, but did not address the reliability of the system. Ubertini (2010) studied the same problem deepening, in particular, the effects of frequency detuning and irregular distributions of the mechanical parameters of TMDs. This study showed that frequency detuning of TMDs in MTMDs configurations can highly improve control performance in a probabilistic perspective and that an irregular distribution of the added mass can increase control robustness. However, while uncertainties were indirectly considered in the formulation, the study did not address the probability of failure.

To the best knowledge of the authors, reliability studies via non-deterministic analysis against multimode flutter in literature has been only conducted for uncontrolled bridges, and never extended to bridges equipped with TMDs. In this paper, we present a reliability-based design methodology for designing a TMD system for suppression of bridge flutter. The main contribution of this paper resides in a formulation that enables the reliability-based design of MTMDs for bridge flutter suppression, and the demonstration of the superiority of the proposed method with respect to a conventional deterministic approach. The procedure is based on a parametric elastodynamic continuum model of the structural system and on a classical first-order method for computing the reliability index (Nowak and Collins 2000; Saydam and Frangopol 2013). It provides means to compute flutter reliability for a bridge equipped with an arbitrary configuration of TMDs. A parametric investigation is presented in order to assess the performance of the proposed design methodology against a classic deterministic design approach, which relies on nominal values of mechanical and aerodynamic parameters. The investigation is numerically conducted on single TMDs and MTMDs of similar and different tuning frequencies. The numerical model is based on the New Carquinez Bridge (NCB), an existing bridge located in CA, USA (Jones and Scanlan 2001; Caracoglia 2008). Two deck cross-sections of different aerodynamic properties are considered.

The rest of the paper is organized as follows. The mathematical formulations for analyzing flutter of long-span bridges equipped with TMDs are presented at first. These include formulations for both the deterministic and non-deterministic methods. Then, the parametric analysis for comparing both analysis methods is introduced. They are conducted on various control cases: 1) uncontrolled; 2) single TMD; 3) MTMD tuned at the same frequency; and 4) MTMD tuned at different frequencies. Analysis results are discussed and the paper is ended with main conclusions.
Mathematical formulation

Deterministic model

Equations of motion Consider the single-span suspension bridge illustrated in Fig. 1 of span $L$ equipped with $n_T$ TMDs (composing an MTMD system) and subjected to transverse wind loading. The structure is composed of two main cables, a stiffening girder (deck) and a uniform distribution of vertical hangers. The generic $i$-th TMD is composed of a pair of mass-spring-damper systems located at opposite edges of the deck at a distance $2l_Ti$.

The main cables are placed at a distance $2b$ and hinged at fixed anchors placed at the same vertical elevation. They are modeled as mono-dimensional linearly elastic continua with negligible flexural and shear rigidities. Their static profile is approximated by a parabolic function, $f$ being the sag and $H$ the static horizontal component of tension in each of the main cables (see Fig. 1 (a)). The deck is modeled as a uniform, linearly elastic beam with Euler-Bernoulli flexural behavior and classic St. Venant torsional behavior and its reference width is denoted by $B$. The hangers are assumed uniformly distributed, massless and inextensible. The Young moduli of the materials constituting the cables and the deck are denoted by $E_c$ and $E_d$, respectively. The shear modulus of the material constituting the deck is denoted by $G_d$. The area moment of inertia of the deck cross-section around the neutral (horizontal) axis is denoted by $I_d$, and its torsional constant is denoted by $J_d$. The motion of the bridge is described by vertical deflection, $v(x, t)$, and twist rotation, $\theta(x, t)$. The vertical displacement and the rotation of the $i$-th TMD are denoted by $q_{vTi}(t)$ and $q_{\theta Ti}(t)$, respectively.

The wind blows in the horizontal cross-deck direction at a speed $U$ and provokes a self-excited lift, $L_{se}(x, t)$, and pitching moment, $M_{se}(x, t)$, per unit length acting on the deck, where $x$ is the axis along the deck and $t$ is time. For simplicity, a uniform wind speed profile is assumed in the formulation. This
provides a more straightforward investigation of reliability aspects in the design of TMDs. However, recent literature showed that non-uniform wind distributions may affect flutter conditions, which should be considered in practical design. The interested reader is referred to (Arena et al. 2014) for more details on this aspect. Self-excited loads are expressed by means of aerodynamic indicial functions $\Phi_{Lv}(t)$, $\Phi_{L\theta}(t)$, $\Phi_{Mv}(t)$ and $\Phi_{M\theta}(t)$ (Kwon and Park 2004; Costa and Borri 2006) as follows:

\[ L_{se} = \frac{1}{2} \rho U^2 B c'_{L} \left( \frac{\dot{v}(x,t)}{U} + \Phi_{L\theta}(0) \theta(x,t) + \int_{0}^{t} \dot{\Phi}_{Lv}(t-\tau) \dot{v}(x,\tau) U d\tau + \int_{0}^{t} \dot{\Phi}_{L\theta}(t-\tau) \theta(x,\tau) d\tau \right) \]

\[ M_{se} = \frac{1}{2} \rho U^2 B^2 c'_{M} \left( \frac{\dot{v}(x,t)}{U} + \Phi_{M\theta}(0) \theta(x,t) + \int_{0}^{t} \dot{\Phi}_{Mv}(t-\tau) \dot{v}(x,\tau) U d\tau + \int_{0}^{t} \dot{\Phi}_{M\theta}(t-\tau) \theta(x,\tau) d\tau \right) \]

where $\rho$ is the air density, $c'_{L}$ and $c'_{M}$ are the derivatives of the lift and moment coefficients of the deck cross-section with respect to the wind angle of attack and evaluated for a zero angle of attack and a dot denotes time differentiation. Indicial functions are approximated through the widely adopted formula (Costa and Borri 2006)

\[ \Phi_{Rr}(t) = 1 - \sum_{i=1}^{N_{Rr}^R} a_{i}^{Rr} \cdot \exp \left( -b_{i}^{Rr} \frac{2U}{B} t \right) \]

with $R = \{ L, M \}$, $r = \{ v, \theta \}$ and where $a_{i}^{Lv}$, $a_{i}^{L\theta}$, $a_{i}^{Mv}$, $a_{i}^{M\theta}$, $b_{i}^{Lv}$, $b_{i}^{L\theta}$, $b_{i}^{Mv}$ and $b_{i}^{M\theta}$ are coefficients typically determined from wind tunnel tests through fitting of measured aeroelastic derivatives (Costa and Borri 2006; Salvatori and Borri 2007) and $N_{R}^{Lv}$, $N_{R}^{L\theta}$, $N_{R}^{Mv}$ and $N_{R}^{M\theta}$ are numbers selected to truncate the series in (3) at the desired accuracy.

Partial differential equations governing vertical and torsional motion of the bridge are taken at first in the time domain using the continuum model presented in (Materazzi and Ubertini 2011; Ubertini 2014) and introducing aeroelastic loads given by Eqs. (1,2). The obtained equations are then transformed conveniently in the modal space. After straightforward computations, the equations of motion in terms of the $i$-th vertical and torsional modal amplitudes, denoted as $q_{i}^{v}(t)$ and $q_{i}^{\theta}(t)$, respectively, are written
in the same general form as

\[ \ddot{q}_i^v(t) + 2\xi_i^v\omega_i^v \dot{q}_i^v(t) + \omega_i^v^2 q_i^v(t) = \frac{1}{2} \rho U^2 B b^v c_i^b c_{R_r} \left( \left( 1 - \sum_{j=1}^{N_{R_r}} a_{j,R_r} \right) \frac{d^{cr}q_i^r(t)}{dt^{cr}} + \left( 1 - \sum_{j=1}^{N_{R_r}} a_{j,R_r} \right) \sum_{k=1}^{N_{R_r}} \frac{d^{cs}q_k^s(t)}{dt^{cs}} + \sum_{j=1}^{N_{R_r}} \sum_{k=1}^{N_{R_r}} \varphi_{ik}^r \varphi_{jk}^s R_{R_r,s_r} \right) + \sum_{j=1}^{N_r} \sum_{k=1}^{N_s} \left( \omega_{Tk}^v \mu_{Tik}^v (q_{Tk}^r - \sum_{j=1}^n \phi_j^v (x_{Tk}) q_j^r(t)) + 2\xi_{Tk}^v \omega_{Tk}^v \mu_{Tik}^v (\dot{q}_{Tk}^r - \sum_{j=1}^n \phi_j^v (x_{Tk}) \dot{q}_j^r(t)) \right) \]

where \( \phi^v_i(x) \) and \( \phi^\theta_i(x) \) are vertical and torsional eigenfunctions of the bridge, respectively, \( x_{Ti} \) is the location of the \( i \)-th TMD and \( m^v \) and \( m^\theta \) are vertical and torsional bridge masses per unit length, respectively. Closed-form expressions for \( \phi^v_i(x) \) and \( \phi^\theta_i(x) \), as well as for the corresponding natural circular frequencies, \( \omega_i^v \) and \( \omega_i^\theta \), can be found in (Materazzi and Ubertini 2011; Ubertini 2014). Vertical and torsional motions of the \( k \)-th TMD are written as

\[ \ddot{q}_Tk(t) + 2\xi_{Tk}^v \omega_{Tk}^v (\dot{q}_{Tk}^v(t) - \sum_{j=1}^n \phi_j^v (x_{Tk}) q_j^v(t)) + \omega_{Tk}^v^2 (q_{Tk}^v(t) - \sum_{j=1}^n \phi_j^v (x_{Tk}) q_j^v(t)) = 0 \]

where \( r = \{ v, \theta \} \), \( s_v = \theta, s_\theta = v \), \( b_v = 1, b_\theta = 2 \), \( c_v = 1, c_\theta = 0 \), \( R_v = L, R_\theta = M \). In Eqs. (4,5) \( \xi^v_i \) and \( \xi^\theta_i \) are vertical and torsional modal damping ratios of the bridge, respectively, while \( \omega_{Tk} \) and \( \xi_{Tk} \) are circular frequency and damping ratio of the \( k \)-th TMD, respectively. The following integral factors have been also introduced:

\[ \varphi_{ij}^{rs} = \frac{\int_0^1 \phi_i^v (x) \phi_j^s (x) dx}{\int_0^1 \phi_i^v (x)^2 dx}, \quad \mu_{Tik}^r = \frac{m_{Tk}^v}{m^r} \frac{\int_0^1 \phi_i^v (x_{Tk}) dx}{\int_0^1 \phi_i^v (x)^2 dx} \]

with \( \{ r, s \} = \{ v, \theta \} \), \( m^v_{Ti} \) and \( m^\theta_{Ti} = m^v_{Ti} l_{Ti}^2 \) being the inertial and torsional masses of the \( i \)-th TMD, respectively.

Memory terms \( z_{ij}^{R_r}(t) \) in Eq. (4) are defined as

\[ z_{ij}^{R_r}(t) = \frac{2a_{ij,R_r} B^2 c_{R_r}}{U} \int_0^t \exp(-b_{ij,R_r}^2 U (t - \tau)) \frac{d^{cr}q_i^r(t)}{dt^{cr}} \ d\tau \]
with \( R = \{ L, M \} \) and \( r = \{ v, \theta \} \). By differentiating Eq. (7) with respect to time and using Leibnitz’s rule, the following first order equation is obtained:

\[
\dot{z}_{ij}^{Rr}(t) = \frac{2b_j^{Rr} U}{B} \left( \frac{a_j^{Rr}}{U_c^r} \frac{d^c q_i^r(t)}{dt^c} - z_{ij}^{Rr}(t) \right)
\]  (8)

Eqs. (4,5,8) are rewritten in the state-space:

\[
\dot{x}(t) = A(U)x(t)
\]  (9)

where \( A(U) \) is the system matrix and \( x(t) \) is the state vector containing modal amplitudes, TMD generalized displacements, their time derivatives and memory terms, Eq. (7), that represent additional aerodynamic state variables of the system. The expression of matrix \( A(U) \) can be readily deduced from Eqs. (4,5,8) and is not reported here for brevity.

**Multimode flutter analysis procedure** An harmonic solution for Eq. (9) with the form \( x = x_0 \exp(i\omega t) \), where \( i = \sqrt{-1} \) and \( x_0 \) is a vector of amplitude coefficients, is sought. Substituting in Eq. (9) yields to the following eigenvalue problem

\[
(A(U) - i\omega I)x_0 = 0
\]  (10)

The minimum positive wind speed, \( U_c \), for which Eq. (10) is satisfied with \( x_0 \neq 0 \) is the critical flutter wind speed, while the corresponding (real) eigenvalue, \( \omega = \omega_c \), is the critical circular frequency.

The eigenvalue problem in Eq. (10) is solved numerically as it is typically done in the literature (Simiu and Scanlan 1986). The eigenvalues of matrix \( A(U) \) are computed for different values of the wind speed \( U \). The motion is stable until all complex conjugate pairs of eigenvalues have negative real parts, while coupled flutter occurs when a purely imaginary complex conjugate pair of eigenvalues appears. Therefore, the critical wind speed, \( U = U_c \), is iteratively computed by imposing that the largest real part of the eigenvalues corresponding to coupled structural modes, \( \lambda_{U,i} \) \((i = 1, 2, \ldots, 2n)\), 2\( n \) being the number of retained vertical and torsional modes, is close to zero within a given tolerance \( \text{tol}_\lambda \) (here taken as \( 10^{-5} \)):

\[
|\max_i(\text{Re}(\lambda_{U,i}))| \leq \text{tol}_\lambda
\]  (11)

In order to avoid selection of a higher order critical solution, a penalty of \( 10^3 \) is added to the real part of \( \lambda_{U,i} \) if it is greater than zero. Thus, the system is stable when all damping ratios, \( \xi_{U,i} = -\text{Re}(\lambda_{U,i})/|\lambda_{U,i}| \), of the coupled modes are greater than zero. When the critical condition is attained, the critical flutter
circular frequency is simply given by the imaginary part of the critical eigenvalue $\lambda_{U_c,c}$:

$$\omega_c = \text{Im}(\lambda_{U_c,c}) \quad (12)$$

while the critical eigenvector, $x_{0,c}$, is the (complex) eigenvector of matrix $A(U_c)$ corresponding to $\lambda_{U_c,c}$. Vector $x_{0,c}$ contains the information on the participation of the different structural modes to the coupled flutter mode in terms of amplitude and phase.

**Non-deterministic model**

The flutter problem is governed by large uncertainties arising from the physical and mechanical quantities that are random in nature. In this section, a procedure for flutter reliability analysis of suspension bridges with an arbitrary configuration of TMDs is presented to address these uncertainties. It is based on the above-described numerical computation of the critical flutter speed, which is iterated within a first order reliability analysis procedure. In what follows, the algorithm for reliability analysis is described and random variables listed.

**Algorithm for Reliability Analysis** From the results presented above, one can compute the flutter limit state function $g$ of a bridge equipped with TMDs:

$$g(y, \tilde{U}) = U_c(y) - \tilde{U} \quad (13)$$

where $y$ is the $p$-dimensional vector of random variables characterizing the mechanical system and the aeroelastic properties of the deck and $\tilde{U}$ is the extreme wind speed at the bridge location, which is also a random variable. According to Eq. (13), the failure surface is given by $g(y, \tilde{U}) = 0$.

Random variables $y_i$, contained in vector $y$, are generally non-normally distributed and need to be transformed into equivalent normal random variables. This is done through the well-known Rackwitz-Fiessler approximation (Rackwitz and Fiessler 1978). The same transformation is applied to $\tilde{U}$ to obtain a $(p + 1)$-dimensional vector $Z$, containing equivalent normal random variables in the normalized space. The vector $Z$ contains the transformation of $y$ and the transformed extreme wind speed.

The reliability index, $\beta$, is defined as the minimum distance, in the space of normalized variables, between the origin, $Z = 0$, and the failure surface $g(Z) = 0$ and is iteratively calculated as follows (Hasofer and Lind 1974):

$$\beta^k = v^k T Z^k \quad (14)$$

where $k$ denotes the iteration number and $v^k = -\nabla g(Z^k)T / ||\nabla g(Z^k)||$ is the unit gradient vector of the limit state function in the space of normalized variables that is computed by central finite differences.
The design point is updated at every $k$-th iteration as follows:

$$Z_{i}^{k+1} = v_{i}^{k} \beta^{k} \text{ for } i = 1, 2, \ldots, p$$

$$Z_{p+1}^{k+1} \text{ computed from } g(Z^{k+1}) = 0$$

(15)

until the following conditions are satisfied:

$$\left| \frac{\beta^{k} - \beta^{k-1}}{\beta^{k}} \right| \leq \epsilon_{\beta}$$

$$\left| \frac{(y^{k+1} - y^{k})}{y^{k}} \right| \leq \epsilon_{y}$$

(16)

where $./$ denotes the element-wise division operator, while $\epsilon_{\beta}$ and $\epsilon_{Z}$ are small user-defined tolerances. Here, such tolerances are both taken as 0.01, which results from a trade-off between computational speed and accuracy, under the assumption that a 1% relative errors in the computation of $\beta$ and $y$ are acceptable. It is worth noting that the second expression in Eq. (15) ensures that $Z^{k}$ belongs to the failure surface. As observed in other literature works on flutter reliability analysis, for instance in (Ge et al. 2000), this improves the accuracy of the method in approximating the reliability index in presence of a limit state function being non-linear in terms of basic random variables.

The presented algorithm may exhibit convergence issues depending upon the initial guess on $Z$. When convergence is not achieved in the first few iterations, the algorithm may diverge. In those cases, a numerical damping parameter $\kappa$ is adopted to enlarge the convergence domain, which is an application of the damped Newton’s method (Goldstein 1962). The following updating rule is adopted starting from the second iteration ($k > 1$):

$$Z_{1:p}^{k+1} = Z_{1:p}^{k} + \kappa \Delta Z_{1:p}^{k+1}$$

(17)

with:

$$\Delta Z_{1:p}^{k+1} = v_{1:p}^{k} \beta^{k} - Z_{1:p}^{k}$$

(18)

where $Z_{1:p}^{k+1}$ are the first $p$ elements of vector $Z$ computed at the $(k + 1)$-th iteration. Note that here, good convergence results are obtained for $\kappa = 0.8$.

The point $Z^{*}$ to which the system converges in the variable space is commonly termed *most probable failure point* or *design point*. The corresponding value of the reliability index, $\beta^{*}$, gives a direct measure of safety, as the probability of failure, $p_{\text{fail}}$, which by definition is the probability that the wind speed reaches the critical value ($\tilde{U} = U_{c}$ or, equivalently, $g(Z) = 0$) can be estimated as $p_{\text{fail}} = \Phi_{0,1}(-\beta^{*})$ (Hasofer and Lind 1974), where $\Phi_{0,1}$ is the standard normal cumulative density function.
Random variables A total of 20 random variables are considered for the problem under investigation: 2 dynamic derivatives, 8 indicial functions coefficients, 2 mass parameters, 3 elastic moduli, 2 stiffness parameters, 2 damping ratios and 1 extreme wind speed. Dynamic derivatives, $c'_L$ and $c'_M$, indicial function coefficients, $a_{tr}^{Rr}$, $R = \{L, M\}$ $r = \{v, \theta\}$, as well as bridge properties $m^v$, $m^\theta$, $E_c$, $E_d$, $I_d$, $G_d$, $J_d$ are modeled as normal random variables. Structural modal damping ratios $\xi^v_i$ and $\xi^\theta_i$ are modeled as Lognormal random variables. Extreme wind speed is modeled through a Gumbel distribution with mean $\mu_U$ and standard deviation $\sigma_U$, as suggested in literature (Simiu et al. 2001).

Remark: the adopted model of aeroelastic loads based on indicial functions (Eqs. (1,2,3)) allows computational efforts to remain within reasonable limits. In the computational reliability analysis algorithm presented above, uncertain parameters are varied several times and, at each iteration step, flutter analysis is performed $2(p + 1)$ times in order to compute the gradient of the limit state function via central finite differences. While multimode flutter analysis based on indicial functions involves the solution of a typical eigenvalue problem of a linear autonomous system (Eq. (10)) this analysis based on aeroelastic derivatives would be computationally more demanding. In that case, matrix $A$ would depend on both $U$ and $\omega$, resulting in a nonlinear eigenvalue problem and the requirement of expensive numerical procedures to compute $U_c$ and $\omega_c$. It follows that the computational burden would become substantial when using aeroelastic derivatives.

In the present study, both deterministic and non-deterministic models and all related calculations are implemented in computational codes developed by the authors in MATLAB (The MathWorks 2012).

Parametric Analysis

In this section, the presented deterministic and non-deterministic models for performance evaluation of TMDs against multimode bridge flutter are applied to a bridge example and compared.

Simulated Control Cases

Four different control strategies are considered: 1) uncontrolled; 2) single TMD; 3) MTMD tuned at the same frequency; and 4) MTMD tuned at different frequencies. Table 1 summarizes the considered analysis control cases. The number, $n_T$, mass ratios, $m^v_{T_i}/(m^v L)$, and positions, $x_{T_i}$, of TMDs are reported in the table. U1 denotes the uncontrolled case, S1-4 the four single TMD cases, M1-3 the three MTMD cases tuned at the same frequency, and MC1-3 the three MTMD cases tuned at different frequencies. Note that the presented case studies, while not necessarily optimal on a technical or design perspective, are intended to be used as benchmark cases to demonstrate the advantages of the proposed reliability-based design methodology over a conventional deterministic approach.
Table 1. Control analysis cases (\(1_n\) denotes an \(n\)-dimensional row vector with all unit elements)

<table>
<thead>
<tr>
<th>Case</th>
<th>(n_T)</th>
<th>(m_{T1}^v/(m^vL))</th>
<th>(x_{T1}/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>S1</td>
<td>1</td>
<td>0.01</td>
<td>1/4</td>
</tr>
<tr>
<td>S2</td>
<td>1</td>
<td>0.01</td>
<td>1/3</td>
</tr>
<tr>
<td>S3</td>
<td>1</td>
<td>0.01</td>
<td>5/12</td>
</tr>
<tr>
<td>S4</td>
<td>1</td>
<td>0.01</td>
<td>1/2</td>
</tr>
<tr>
<td>M1</td>
<td>3</td>
<td>0.01 [\begin{bmatrix} 1/3 &amp; 1/3 &amp; 1/3 \ 1/3 &amp; 1/3 &amp; 1/3 \ 1/3 &amp; 1/3 &amp; 1/3 \end{bmatrix} ]</td>
<td>[\begin{bmatrix} 1/3 &amp; 1/3 &amp; 1/3 \end{bmatrix} ]</td>
</tr>
<tr>
<td>M2</td>
<td>3</td>
<td>0.01 [\begin{bmatrix} 1/3 &amp; 1/3 &amp; 1/3 \ 1/3 &amp; 1/3 &amp; 1/3 \ 1/3 &amp; 1/3 &amp; 1/3 \end{bmatrix} ]</td>
<td>[\begin{bmatrix} 1/3 &amp; 1/3 &amp; 1/3 \end{bmatrix} ]</td>
</tr>
<tr>
<td>M3</td>
<td>5</td>
<td>0.01 [\begin{bmatrix} 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 \ 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 \ 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 \end{bmatrix} ]</td>
<td>[\begin{bmatrix} 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 &amp; 1/5 \end{bmatrix} ]</td>
</tr>
<tr>
<td>MC1</td>
<td>5</td>
<td>0.01 [\begin{bmatrix} 1/5 \ 1/5 \ 1/5 \ 1/5 \ 1/5 \end{bmatrix} ]</td>
<td>[\begin{bmatrix} 1/5 \ 1/5 \ 1/5 \ 1/5 \ 1/5 \end{bmatrix} ]</td>
</tr>
<tr>
<td>MC2</td>
<td>15</td>
<td>0.01 [\begin{bmatrix} 1/15 \ 1/15 \ 1/15 \ 1/15 \ 1/15 \end{bmatrix} ]</td>
<td>[\begin{bmatrix} 1/15 \ 1/15 \ 1/15 \ 1/15 \ 1/15 \end{bmatrix} ]</td>
</tr>
<tr>
<td>MC3</td>
<td>25</td>
<td>0.01 [\begin{bmatrix} 1/25 \ 1/25 \ 1/25 \ 1/25 \ 1/25 \end{bmatrix} ]</td>
<td>[\begin{bmatrix} 1/25 \ 1/25 \ 1/25 \ 1/25 \ 1/25 \end{bmatrix} ]</td>
</tr>
</tbody>
</table>

U: uncontrolled;  
S: single TMD;  
M: MTMD tuned at the same frequency;  
MC: MTMD tuned at different frequencies.

For the deterministic approach, the performance metric of the control system is the increase in critical wind speed (assumed as a deterministic quantity) achieved by the installation of TMDs. For the non-deterministic approach, the performance metric is the reduction in the probability of flutter occurrence. The comparison between both models is conducted using these performance measures.

Model Assumptions

The numerical simulations are based on the NCB. The mechanical properties of the NCB assumed in this study are summarized in Table 2. The modal parameters of the bridge obtained by the analytical model are derived as described in (Materazzi and Ubertini 2011; Ubertini 2014) and summarized in Table 3 and Fig. 2. Symmetric vertical modes are denoted as V1S, V2S, etc., while antisymmetric ones are denoted as V1A, V2A, etc. Similarly, symmetric torsional modes are denoted as T1S, T2S, etc., while antisymmetric torsional modes are denoted as T1A, T2A, etc. Modal damping ratios equal to 0.3% are assigned to all structural modes.

Aerodynamic and indicial function coefficients corresponding to rectangular cross-sections of width to height ratio \(B/D = 12.5\) (termed R12) and \(B/D = 5\) (termed R05) are taken for the aeroelastic stability analysis. R12 and R05 are representative of streamlined and bluff cross-sections, respectively, and their aerodynamic and aeroelastic properties can be found in Ref. (Costa and Borri 2006) and are summarized in Table 4.
Figure 2. Spectrum of eigenmodes of the NCB: vertical modes (a), torsional modes (b)

Parameters of the Weibull distribution for the extreme wind speed have a mean value $\mu_U = 25 \text{ m/s}$ and a standard deviation $\sigma_U = 6.25 \text{ m/s}$. Coefficients of variation for damping ratios are taken as 40% (Cheng et al. 2005). For simplicity, it is assumed that all vertical and torsional modes have the same damping ratio. Coefficients of variation of inertial and torsional masses of the girder, and flexural moments of inertia and torsional constant of the girder are taken as 10% (Cheng et al. 2005). Coefficients of variations of elastic moduli of cables and girder and shear modulus of the girder are taken as 5%. This value is larger than the expected coefficient of variation of structural steel, which is motivated by the need for accounting for additional uncertainties in global stiffness of structural members. Condensing global stiffness uncertainties into the variability of Young and shear moduli of the material constituting the deck also justifies the assumption of neglecting the mutual relationship between these two coefficients that characterizes isotropic elastic materials.

A coefficient of variation of 15% is taken for the uncertainties in aeroelastic properties, as typically assigned to aeroelastic derivatives (Caracoglia 2008). The equivalence between aeroelastic derivatives and indicial functions shows that vertical aeroelastic derivatives are proportional to $c'_L$ and torsional aeroelastic derivatives are proportional to $c'_M$ (Costa and Borri 2006). Thus, coefficients of variation of $c'_L$ and $c'_M$ are also taken as 15%. An additional uncertainty of 5% is assigned to indicial function coefficients $a_{iRr}^r$ and $b_{iRr}^r$, with $R = \{L, M\}$, $r = \{v, \theta\}$, which reduces correlation among aeroelastic derivatives.

In what follows, mean values of the different parameters are assumed as nominal values in deterministic analysis, while uncertainties are accounted for in the non-deterministic analysis.
Table 2. Mechanical properties of the NCB assumed in this study

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ (m)</td>
<td>728.0</td>
<td>$m$ (kg/m)</td>
<td>21568</td>
<td>$H$ (kN)</td>
<td>$8.97 \cdot 10^4$</td>
</tr>
<tr>
<td>$f$ (m)</td>
<td>77.5</td>
<td>$B$ (m)</td>
<td>25.6</td>
<td>$A_c$ (m$^2$)</td>
<td>0.1651</td>
</tr>
<tr>
<td>$E_c$ (N/m$^2$)</td>
<td>$196.5 \cdot 10^9$</td>
<td>$E_d$ (N/m$^2$)</td>
<td>$206.0 \cdot 10^9$</td>
<td>$G_d$ (N/m$^2$)</td>
<td>$76.5 \cdot 10^9$</td>
</tr>
<tr>
<td>$I_m$ (kg$ \cdot $m$^2$/m)</td>
<td>$2.52 \cdot 10^6$</td>
<td>$I_d$ (m$^4$)</td>
<td>1.64</td>
<td>$J_d$ (m$^4$)</td>
<td>4.79</td>
</tr>
</tbody>
</table>

Table 3. Modal parameters of the NCB (V: vertical mode; T: torsional mode; A: antisymmetric mode; S: symmetric mode)

<table>
<thead>
<tr>
<th>mode</th>
<th>$\omega$ (rad/s)</th>
<th>mode</th>
<th>$\omega$ (rad/s)</th>
<th>mode</th>
<th>$\omega$ (rad/s)</th>
<th>mode</th>
<th>$\omega$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1A</td>
<td>0.841</td>
<td>V1S</td>
<td>1.244</td>
<td>T1A</td>
<td>3.494</td>
<td>T1S</td>
<td>2.782</td>
</tr>
<tr>
<td>V2A</td>
<td>1.968</td>
<td>V2S</td>
<td>1.929</td>
<td>T2A</td>
<td>6.988</td>
<td>T2S</td>
<td>5.304</td>
</tr>
<tr>
<td>V3A</td>
<td>3.554</td>
<td>V3S</td>
<td>2.740</td>
<td>T3A</td>
<td>10.482</td>
<td>T3S</td>
<td>8.747</td>
</tr>
<tr>
<td>V4A</td>
<td>5.676</td>
<td>V4S</td>
<td>4.554</td>
<td>T4A</td>
<td>13.977</td>
<td>T4S</td>
<td>12.234</td>
</tr>
<tr>
<td>V5A</td>
<td>8.363</td>
<td>V5S</td>
<td>6.951</td>
<td>T5A</td>
<td>17.471</td>
<td>T5S</td>
<td>15.726</td>
</tr>
</tbody>
</table>

Table 4. Dynamic derivatives and indicial function (IF) coefficients of R12 and R05 rectangular cross sections

<table>
<thead>
<tr>
<th>IF</th>
<th>$a_i^{RE}$</th>
<th>$b_i^{RE}$</th>
<th>IF</th>
<th>$a_i^{RE}$</th>
<th>$b_i^{RE}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_{LV}$</td>
<td>0.9711</td>
<td>2.146</td>
<td>$\Phi_{LV}$</td>
<td>1.0147</td>
<td>0.7437</td>
</tr>
<tr>
<td>$\Phi_{L\theta}$</td>
<td>1.0218</td>
<td>0.6636</td>
<td>$\Phi_{L\theta}$</td>
<td>0.9589</td>
<td>0.5770</td>
</tr>
<tr>
<td>$\Phi_{MV}$</td>
<td>0.2036</td>
<td>19.5221</td>
<td>$\Phi_{MV}$</td>
<td>[4.5639, $-3.4781$]</td>
<td>[0.3838, 1.7768]</td>
</tr>
<tr>
<td>$\Phi_{M\theta}$</td>
<td>0.9535</td>
<td>2.0876</td>
<td>$\Phi_{M\theta}$</td>
<td>[6.6924, $-5.6613$]</td>
<td>[0.3688, 0.8585]</td>
</tr>
</tbody>
</table>

Control Strategy: Uncontrolled (U1)

Results from the deterministic multimode flutter analysis of the uncontrolled structure (U1) are presented in Fig. 3. Critical uncontrolled flutter wind speeds of $U_c^{R05} = 45.5$ m/s and $U_c^{R12} = 79.9$ m/s are obtained for the R05 and R12 cross-sections, respectively. The cross-section of the existing NCB has an aerodynamic behavior similar to that of the R12 cross-section. The critical wind speed value here obtained for the R12 cross-section agrees well with the one reported for the NCB at the design stage of 74 m/s (Jones and Scanlan 2001). Note that $U_c^{R05} < U_c^{R12}$ because the R05 cross-section is much more sensitive to wind loads compared to R12 due to its bluff shape. For both R05 and R12, the critical flutter mode is the result from a coupling between modes V2S and T1S. The shapes of the critical flutter modes computed for increasing the model order are shown in Fig. 3.

The flutter reliability of the uncontrolled structure is examined by varying the number of modes retained in the model. Results are presented in Figure 4. The flutter reliability index of the bridge with R12 cross-section is equal to $\beta^{R12} = 4.0$, while that of the R05 cross-section is equal to $\beta^{R05} = 2.3$. In terms of
probability of failure, these values correspond to $p_{\text{fail}}^{R12} = 3.4 \cdot 10^{-5}$ and $p_{\text{fail}}^{R05} = 1.0 \cdot 10^{-2}$, respectively. There is a significant difference in reliability of the structure between the streamlined and bluff cross-sections. Another feature in the results is that the convergence of the reliability index is achieved using a few iterations only (see Figure 4 (b) and (c)). This fast convergence is justified by the local regularity of the limit state function which, in the uncontrolled case, approaches linearity for the $R05$ cross-section, where only two iterations are necessary to achieve convergence. This is achieved by taking the mean values as initial guesses for all random variables except for the wind speed, for which the initial guess is taken as equal to the critical flutter wind speed.

Figure 5 shows the evolution of $\beta$ and computational time by varying the tolerances $\epsilon_\beta$ and $\epsilon_y$, Eq. (16), using a core i7 3.4 GHz standard PC. These results demonstrate that values of $\epsilon_\beta = \epsilon_y = 0.01$ provide convergence with minimal computational time.
Figure 4. Flutter reliability index $\beta$ and probability of failure $p_{\text{fail}}$ for the uncontrolled bridge by varying the number of retained modes (a); reliability index versus number of iterations for 20 modes, considering R12 (b) and R05 deck types.

Table 5. Largest sensitivity coefficients of reliability index with respect to mean values and coefficient of variations of random variables (CV stands for coefficient of variation)

<table>
<thead>
<tr>
<th></th>
<th>$a_1^{M\theta}$</th>
<th>$b_1^{M\theta}$</th>
<th>$m$</th>
<th>$E_c$</th>
<th>$G_d$</th>
<th>$J_d$</th>
<th>$\bar{U}$</th>
<th>$CV_{\bar{U}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R12</td>
<td>20.2</td>
<td>-17.8</td>
<td>59.8</td>
<td>17.1</td>
<td>27.4</td>
<td>27.4</td>
<td>-30.7</td>
<td>-45.1</td>
</tr>
<tr>
<td>R05</td>
<td>268.8</td>
<td>204.8</td>
<td>24.6</td>
<td>41.0</td>
<td>31.9</td>
<td>31.9</td>
<td>-84.4</td>
<td>-57.5</td>
</tr>
</tbody>
</table>

The reliability analysis results are further investigated through a sensitivity analysis to evaluate the influence of the different random variables on the reliability index. Results are summarized in Table 5, where only the parameters resulting in a sensitivity coefficient with an absolute value greater than 30 for either the R05 or R12 cases have been considered. Sensitivity coefficients are expressed in percentage as the ratios between relative variation in $\beta$ and relative variation in the parameters. These results demonstrate that torsional aerodynamic parameters strongly affect the reliability index in the case of the R05 cross-section due to the occurrence of a torsionally-driven flutter type instability. On the other hand, the largest sensitivity coefficient in the case of the R12 deck type is the mass $m$, which is related to the bending-driven nature of the flutter instability. In both the R05 and R12 cases, a significant sensitivity of $\beta$ with respect to the mean value of the extreme wind speed and its coefficient of variation is also evidenced, where the negative sign indicates a reduction in reliability for increasing wind speed.

Control Strategy: Single TMD (S1-4)

The case of a single TMD (case S1 in Table 1) is investigated. Figure 6 presents the results of the deterministic flutter analysis, where the initial tuning frequency and damping ratio of the TMD are
Evolution of $\beta$ and computational time by varying tolerances $\epsilon_\beta$ and $\epsilon_\gamma$. Evolution of $\beta$ for R12 (a) and R05 (b) deck types; computational time for R12 (c) and R05 (d) deck types.

arbitrarily selected as $\omega_{T,0} = \omega_c/(1 + \mu)$ and $\xi_{T,0} = \sqrt{\mu/(1 + \mu)}$, where $\mu = m_T^v/(m^v L)$ is the mass ratio of the TMD.

Figures 6 (a) and (b) show the damping ratios of the coupled modes becoming unstable for the R12 and R05 cross-sections, respectively. These damping ratios are for first order (symmetric) and second order (antisymmetric) uncontrolled critical flutter modes and for first order controlled flutter mode versus wind speed. Figures 6 (c) and (d) show the root loci of both uncontrolled and controlled systems for increasing wind-speed.

The results of Figure 6 confirm the ability of the TMD to significantly increase the critical wind speed. In particular, the first order critical wind speed for the R12 cross-section is equal to 107.72 m/s (34.8% higher than uncontrolled), while that for the R05 cross-section is equal to 58.01 m/s (27.5% higher than uncontrolled).

Repeating the same analysis on Figure 6 using the non-deterministic model yields $\beta^{R12} = 4.97$ and $\beta^{R05} = 2.96$. Corresponding probabilities of failure are $p_{fail}^{R12} = 3.3 \cdot 10^{-7}$ and $p_{fail}^{R05} = 1.5 \cdot 10^{-3}$. The TMD reduces $p_{fail}$ by two orders of magnitude in the case of the R12 cross-section and by less than one order of magnitude in the case of the R05 cross-section.

Comparing results from both approaches, the addition of a TMD at mid-span results in increases in critical wind speed and reliability index that are comparable for both R12 and R05 cases (both the
critical wind speed and reliability index in the two cases are increased by approximately 30%). However, the resulting reductions in probability of failure are substantially different, because it is an exponential function of $\beta$. One can conclude that the increase in critical wind speed obtained by installing a control system is weakly correlated with the increase in safety.

The effect of a change in the position of the TMD (cases S2, S3 and S4 in Table 1) and of an increase of its mass ratio, $\mu$, are investigated. Results presented in Figure 7 show that S2 gives the best result for both bridge decks and analysis methods. This location ($L/3$) is consistent with the mode shape of the critical mode. In the deterministic cases, the critical flutter wind speed nonlinearly increases with $\mu$ except for those cases where a second order critical mode arises that is not controllable with the TMD, because it is located at a node of the critical mode (e.g. case S4 in the R05 case). This is a notable difference with the non-deterministic analysis, where the probability of failure decreases with increasing $\mu$ for all analysis cases. A possible explanation is that the most probable failure point corresponds to a wind speed

---

**Figure 6.** Critical flutter conditions for the bridge example: damping ratio of critical flutter mode for R12 (a) and R5 (b) deck types (black lines denote first-order (full) and second-order (dashed) uncontrolled critical flutter modes, the blue line (full-circle) denotes first order controlled flutter mode (case S1)); root loci of the system for varying wind speed for R12 (c) and R05 (d) deck types (uncontrolled in black, controlled in blue).
Figure 7. Deterministic and non-deterministic flutter analysis for the bridge example with one TMD at different positions and with increasing mass ratio $\mu$: nominal critical wind speed for R12 (a) and R05 (b) deck types; probability of failure for R12 (c) and R05 (d) deck types.

Figure 8. Number of iterations and computational time for analyzing one TMD at different positions and with increasing mass ratio $\mu$: number of iterations for R12 (a) and R05 (b) deck types; computational time for R12 (c) and R05 (d) deck types.
Figure 9. Deterministic and non-deterministic flutter analysis for the bridge example with one TMD placed at $x_T = L/3$ (case S2): critical wind speed as a function of frequency tuning and damping ratio for R12 (a) and R05 (b) deck types; probability of failure as a function of frequency tuning and damping ratio for R12 (c) and R05 (d) deck types.

lower than the one corresponding to the second-order instability. Results also suggest that augmenting the mass ratio $\mu$ is much more effective from a probabilistic standpoint compared with the deterministic approach. For example, changing the mass ratio from $\mu = 0.01$ to $\mu = 0.02$ results in 11.4% and 12.3% increases in the critical wind-speed for a TMD installed at $L/3$ (S2) for the R12 and the R05 deck types, respectively (deterministic approach), while the probability of failure is decreased by 91% and 62.6% with the non-deterministic approach, respectively.

Figure 8 shows number of iterations and computational time required for computing $\beta$ values in the analysis of Figure 7 using a core i7 3.4 GHz standard PC. In order to achieve fast convergence, the initial guess of the design is taken as the result of the previous analysis with a smaller TMDs mass ratio. This way, convergence is achieved in 2 or 3 iterations with a reasonable computational time. It is also shown that computational time is not proportional to the number of reliability iterations, because it depends on the number of iterations that are necessary for each flutter analysis conducted within the reliability loop.

The design of a single TMD system with the optimal location found above ($L/3$) and a mass ratio $\mu = 0.01$ can be further refined by altering its stiffness and damping properties. Results for different
tuning frequencies and damping ratios are plotted in Figure 9 in terms of critical wind speed and probability of failure. The optimal design parameters, obtained by means of a direct full-domain search, are shown by a red dot. The optimal frequency ratio is the same between both design methodologies, while the optimal damping ratio differs significantly.

**Control Strategy: MTMD tuned at the same frequency (M1-3)**

The deterministic and reliability-based design methodologies are compared for MTMD tuned as the same frequency (same stiffness and damping characteristics). Deterministic critical wind speeds and probabilities of failure are computed for three cases of MTMD (cases M1, M2 and M3 in Table 1) and compared against the optimal S2 case in Table 1.

Tables 6 and 7 summarize the results in terms of: optimal deterministic ($\omega_{T,\text{opt}}^d$, $\xi_{T,\text{opt}}^d$) and probabilistic ($\omega_{T,\text{opt}}^p$, $\xi_{T,\text{opt}}^p$) design parameters, nominal value of critical wind speed obtained for the optimal deterministic ($U_c^d$) and probabilistic ($U_c^p$) design parameters, and probability of failure corresponding to the optimal deterministic ($p_{\text{fail}}^d$) and probabilistic ($p_{\text{fail}}^p$) design parameters. A metric $\tilde{G}_p$ is also created to compare the probability of failure between deterministic based design and probabilistic based design. It is calculated in percentage using:

$$\tilde{G}_p = \left(1 - \frac{p_{\text{fail}}^p}{p_{\text{fail}}^d}\right) \cdot 100 \tag{19}$$

Results from Table 6 show optimal design parameters that are similar to results from Figure 9. Also, the optimal frequency tuning is in good agreement between both the deterministic and non-deterministic design approaches, while the optimal damping tuning is not, as found above. Results from Table 7 show that the probabilistic design guarantees higher levels of safety compared with the deterministic approach for all of the MTMD analysis cases. The results also show that the deterministic approach does not succeed at selecting an optimal design case, exhibiting similar performance on $U_c^d$ for all of the options (M1-3), while the non-deterministic approach suggests better performance on $p_{\text{fail}}^p$ using design cases M2 or M3.

**Control Strategy: MTMDs with different resonant frequencies**

In this section, the design formulation for MTMD tuned at different frequencies is studied. Such control scheme has the potential to improve robustness with respect to frequency mistuning (Kwon and Park 2004). The nominal critical wind speed and probability of flutter occurrence are examined considering three control cases. These cases are characterized by the same amount of added mass corresponding to a total mass ratio $\mu = 0.01$. The MTMD is composed of clusters of TMDs, where each cluster is
Table 6. Comparison between optimal design parameters for MTMD using deterministic and non-deterministic approaches.

<table>
<thead>
<tr>
<th></th>
<th>$\omega^d_{T,opt}$</th>
<th>$\omega^p_{T,opt}$</th>
<th>$\xi^d_{T,opt}$</th>
<th>$\xi^p_{T,opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2 R12</td>
<td>0.94</td>
<td>0.94</td>
<td>0.83</td>
<td>1.34</td>
</tr>
<tr>
<td>S2 R05</td>
<td>0.96</td>
<td>0.96</td>
<td>0.92</td>
<td>1.06</td>
</tr>
<tr>
<td>M1 R12</td>
<td>0.91</td>
<td>0.91</td>
<td>0.70</td>
<td>1.28</td>
</tr>
<tr>
<td>M1 R05</td>
<td>0.96</td>
<td>0.94</td>
<td>0.74</td>
<td>0.97</td>
</tr>
<tr>
<td>M2 R12</td>
<td>0.91</td>
<td>0.91</td>
<td>0.83</td>
<td>1.28</td>
</tr>
<tr>
<td>M2 R05</td>
<td>0.96</td>
<td>0.94</td>
<td>0.87</td>
<td>0.98</td>
</tr>
<tr>
<td>M3 R12</td>
<td>0.91</td>
<td>0.91</td>
<td>0.83</td>
<td>1.28</td>
</tr>
<tr>
<td>M3 R05</td>
<td>0.96</td>
<td>0.94</td>
<td>0.83</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 7. Comparison between probability of failure obtained through deterministic and probabilistic approaches (nominal critical wind speed values are also reported).

<table>
<thead>
<tr>
<th></th>
<th>$U^d_c$ (m/s)</th>
<th>$U^p_c$ (m/s)</th>
<th>$p^d_{fail}$</th>
<th>$p^p_{fail}$</th>
<th>$\tilde{G}_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2 R12</td>
<td>127.7</td>
<td>115.7</td>
<td>$7.6 \cdot 10^{-8}$</td>
<td>$2.8 \cdot 10^{-8}$</td>
<td>63.2</td>
</tr>
<tr>
<td>S2 R05</td>
<td>61.7</td>
<td>61.2</td>
<td>$6.4 \cdot 10^{-4}$</td>
<td>$6.2 \cdot 10^{-4}$</td>
<td>3.1</td>
</tr>
<tr>
<td>M1 R12</td>
<td>129.8</td>
<td>111.6</td>
<td>$2.4 \cdot 10^{-7}$</td>
<td>$7.5 \cdot 10^{-8}$</td>
<td>68.8</td>
</tr>
<tr>
<td>M1 R05</td>
<td>63.1</td>
<td>58.7</td>
<td>$1.1 \cdot 10^{-3}$</td>
<td>$7.1 \cdot 10^{-4}$</td>
<td>35.5</td>
</tr>
<tr>
<td>M2 R12</td>
<td>129.7</td>
<td>115.8</td>
<td>$7.4 \cdot 10^{-8}$</td>
<td>$3.6 \cdot 10^{-8}$</td>
<td>51.4</td>
</tr>
<tr>
<td>M2 R05</td>
<td>62.5</td>
<td>60.7</td>
<td>$9.9 \cdot 10^{-4}$</td>
<td>$5.3 \cdot 10^{-4}$</td>
<td>46.5</td>
</tr>
<tr>
<td>M3 R12</td>
<td>129.7</td>
<td>115.8</td>
<td>$7.4 \cdot 10^{-8}$</td>
<td>$3.6 \cdot 10^{-8}$</td>
<td>51.4</td>
</tr>
<tr>
<td>M3 R05</td>
<td>63.0</td>
<td>60.5</td>
<td>$1.0 \cdot 10^{-3}$</td>
<td>$5.4 \cdot 10^{-4}$</td>
<td>46.0</td>
</tr>
</tbody>
</table>

composed of five small TMDs tuned at different frequencies. These analysis cases are summarized in Table 1. The tuning frequencies of TMDs belonging to the same cluster are equally spaced between $\omega_T - \epsilon$ and $\omega_T + \epsilon$, $\epsilon$ being a small detuning frequency value:

$$\omega_{Ti} = \omega_T - \epsilon + \frac{2\epsilon}{n_{T,c}-1}(i-1) \quad \text{for } i = 1, 2, \ldots, n_{T,c} \tag{20}$$

where $n_{T,c}$ is the number of TMDs in each cluster (in the present case $n_{T,c} = 5$).

Critical wind speeds and probabilities of flutter occurrence are investigated by varying $\epsilon$ in Eq. (20). A truly optimal design would also require to find optimal values for $\omega_T$ and $\xi_T$. This is beyond the scope of the present investigation. Here, $\omega_T$ and $\xi_T$ are assumed for each analysis case to be equal to $\omega^d_{T,opt}$, $\xi^d_{T,opt}$ and $\omega^p_{T,opt}$, $\xi^p_{T,opt}$ (optimal deterministic and probabilistic values obtained for $\epsilon = 0$), respectively, while $\epsilon$ is varied from 0 to 0.2.

Results summarized in Table 8 show the maximum critical wind speed and the minimum probability of failure obtained by varying $\epsilon$ in all cases. In such a table, the relative gain obtained by varying $\epsilon$ according to the non-deterministic method $\tilde{G}_p^\epsilon$ is calculated in percentage using:
Table 8. Comparison between critical wind speed and probability of failure obtained using MTMDs with different tuning frequencies.

<table>
<thead>
<tr>
<th></th>
<th>( \max (U_c) )</th>
<th>( \min (p_{\text{fail}}) )</th>
<th>( \tilde{G}_p^{\epsilon} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC1 R12 (( \omega_{T,\text{opt}}^p, \xi_{T,\text{opt}}^p ))</td>
<td>115.6</td>
<td>4.3 ( \cdot 10^{-8} )</td>
<td>6.3</td>
</tr>
<tr>
<td>MC1 R12 (( \omega_{T,\text{opt}}^d, \xi_{T,\text{opt}}^d ))</td>
<td>124.5</td>
<td>1.5 ( \cdot 10^{-8} )</td>
<td>72.5</td>
</tr>
<tr>
<td>MC1 R05 (( \omega_{T,\text{opt}}^p, \xi_{T,\text{opt}}^p ))</td>
<td>61.9</td>
<td>4.0 ( \cdot 10^{-4} )</td>
<td>2.7</td>
</tr>
<tr>
<td>MC1 R05 (( \omega_{T,\text{opt}}^d, \xi_{T,\text{opt}}^d ))</td>
<td>63.2</td>
<td>3.7 ( \cdot 10^{-4} )</td>
<td>17.7</td>
</tr>
<tr>
<td>MC2 R12 (( \omega_{T,\text{opt}}^p, \xi_{T,\text{opt}}^p ))</td>
<td>112.6</td>
<td>5.0 ( \cdot 10^{-8} )</td>
<td>6.4</td>
</tr>
<tr>
<td>MC2 R12 (( \omega_{T,\text{opt}}^d, \xi_{T,\text{opt}}^d ))</td>
<td>126.6</td>
<td>1.8 ( \cdot 10^{-8} )</td>
<td>68.8</td>
</tr>
<tr>
<td>MC2 R05 (( \omega_{T,\text{opt}}^p, \xi_{T,\text{opt}}^p ))</td>
<td>60.1</td>
<td>5.4 ( \cdot 10^{-4} )</td>
<td>0.0</td>
</tr>
<tr>
<td>MC2 R05 (( \omega_{T,\text{opt}}^d, \xi_{T,\text{opt}}^d ))</td>
<td>61.7</td>
<td>8.5 ( \cdot 10^{-4} )</td>
<td>34.6</td>
</tr>
<tr>
<td>MC3 R12 (( \omega_{T,\text{opt}}^p, \xi_{T,\text{opt}}^p ))</td>
<td>116.1</td>
<td>3.6 ( \cdot 10^{-8} )</td>
<td>0.0</td>
</tr>
<tr>
<td>MC3 R12 (( \omega_{T,\text{opt}}^d, \xi_{T,\text{opt}}^d ))</td>
<td>126.7</td>
<td>1.6 ( \cdot 10^{-8} )</td>
<td>59.7</td>
</tr>
<tr>
<td>MC3 R05 (( \omega_{T,\text{opt}}^p, \xi_{T,\text{opt}}^p ))</td>
<td>61.9</td>
<td>3.9 ( \cdot 10^{-4} )</td>
<td>1.0</td>
</tr>
<tr>
<td>MC3 R05 (( \omega_{T,\text{opt}}^d, \xi_{T,\text{opt}}^d ))</td>
<td>61.7</td>
<td>8.0 ( \cdot 10^{-4} )</td>
<td>33.3</td>
</tr>
</tbody>
</table>

\[
\tilde{G}_p^{\epsilon} = \left(1 - \min_{p_{\text{fail}}^{\epsilon=0}} (p_{\text{fail}})\right) \cdot 100
\]  

The cases corresponding to the smallest probabilities of failure are investigated in more details in Figure 10. These results reveal that a value of \( \epsilon \) greater than zero (MTMD with different tuning frequencies) can significantly reduce the probability of failure, at least in the R12 case, while almost not affecting deterministic critical wind speed. The probability of failure can be minimized substantially by altering \( \epsilon \).

Finally, it can also be noted that the TMD dynamic displacements at the critical flutter condition can be significantly reduced by adopting multiple configurations. This is demonstrated in Figure 11. These results have been obtained using the mode shapes of the controlled bridge at the critical conditions, as shown in Figures 7 and 10.

**Conclusions**

We have presented a reliability-based design procedure of TMD systems used in the suppression of bridge flutter. The procedure is a computationally efficient method to compute the probability of flutter occurrence based on a continuum formulation of the structural system and a modified first-order method of reliability analysis accounting for the relevant uncertainties affecting the design.

The reliability-based design approach was compared with a deterministic methodology using a continuum elastodynamic model of a suspension bridge. Two different deck types were considered. One with a streamlined deck cross-sectional shape, and one with a bluff shape. Various control strategies
were investigated: single TMDs, MTMDs, and MTMD clusters with different tuning frequencies. The performance of the deterministic design was based on critical wind speed, while the performance of the non-deterministic approach was based on the probability of failure.

Both design methodologies resulted in a similar optimal TMD placement for all design cases under investigation, as well as a similar optimal tuning frequency. However, the non-deterministic approach...
Figure 11. Ratios between vertical modal displacements of windward and leeward TMDs and corresponding deck displacements at the critical flutter condition, in single and multiple tuned mass damper configurations (for single TMDs only the maximum displacements among windward and leeward TMDs are presented using dashed lines): R12 deck type with MC1 (a), MC2 (b) and MC3 (c) control systems; R05 deck type with MC1 (d), MC2 (e) and MC3 control systems (f).

suggested that the increase in the TMD’s mass ratio is significantly more effective at reducing the probability of failure than it was at increasing the critical wind speed with the deterministic approach. Also, it was found that the deterministic design approach does not guarantee the maximum safety.

This conclusion was confirmed with the simulations on the MTMD clusters (control cases MC1-MC3). Using different tuning frequencies within a cluster leads to a substantial increase in performance with the non-deterministic approach, suggesting that such design can be more effective against mistuning. Conversely, this strategy did not result in any significant variation in performance using the deterministic design approach.

The same general conclusions were drew for both bridge deck types. The numerical example used in this paper demonstrates the high potential of a reliability-based design approach for designing TMD control systems for suppressing bridge flutter. It could lead to savings in terms of weight and costs by installing smaller TMDs to achieve a given performance, and provide enhanced safety against uncertainties.
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References


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